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HYPERBOLIC-FM (CHYPE)

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Army Missile Command
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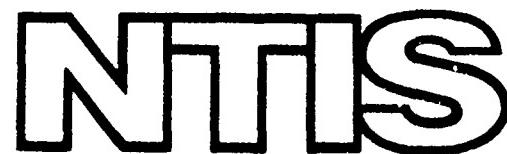
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13. ABSTRACT

This report investigates a hyperbolic frequency modulation technique that is doppler invariant. This doppler invariant property assures its usefulness in both the realms of acoustic echolocation and radar. Bat echolocation techniques are discussed and are shown to be surprisingly analogous to current radar techniques. Spectral characteristics of the compressed hyperbolic-FM are examined with respect to its pulse compression capability in extremely high velocity target environments and is shown to be superior to the conventional linear-FM. Matched filtering is simulated via an 11-bit Fast Fourier Transform.

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1. Introduction

Today, radar and sonar systems depend heavily on pulse compression processing. The use of high time-bandwidth product (T · B) or pulse compression waveforms in these systems has been considerably investigated and refined in the past 10 years. This investigation has for the most part been focused on the linear-FM or chirp modulation where the frequency sweep is linear over the pulse. This is quite natural due to the linear-FM waveform's suitability in illuminating today's class of relatively slow moving target and its ease of generation and application. State of the art delay line technology has satisfactorily established single pulse time-bandwidth products on the order of 1000.

Linear-FM modulation loses its effectiveness, however, when used in a high velocity target environment. A limit is placed on the radar's maximum attainable compression ratio which stems from doppler distortions of the compressed pulse. Put simply, the faster the target, the lower the effective compression ratio. While this may not present extreme difficulties for today's surface based radars due to the "relatively slow" velocities of aircraft and missiles being illuminated, a situation can already be forseen where accurate resolution of satellites or space probes utilizing ultra-high time-bandwidth product signals would be inadequate if illuminated by the familiar linear-FM waveform. Doppler distortions would render this type of modulation somewhat ineffectual.

This report will examine a modulation scheme commonly called hyperbolic-FM or linear period modulation that appears to be well suited for matched filtering in a high target doppler environment. Interestingly enough there is mounting evidence that this modulation could well be that used by hunting bats.

Acoustic orientation by bats and porpoises has been recognized for some time. Spallanzani in 1793 performed experiments with bats that led him to conclude that "the ear of the bat serves more efficiently for seeing, or at least for measuring distances, than do its eyes." Various researchers throughout the years have materially added to the knowledge pertaining to bat echolocation. Griffin's work [1] clearly established the adaptive nature of the bat echolocation technique. While not all species of bats are sightless or employ echolocation, those that are endowed with poor visual perception are typically the ones that have developed extraordinary echolocation capabilities. It seems that this widely misrepresented mammal has implemented for eons a location and ranging technique which man has only recently discovered. In particular, it appears that the modulation of the bat's transmitted signal, which possesses time-bandwidth products significantly greater than unity, is structured such that insect velocities create little if any degradation of the received processed signal. It will become evident that there exist some astonishing parallels between bat and radar/sonar signal processing.

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2. Matched Filters

A filter is said to be matched to a waveform if the filter's impulse response, $h(t)$, is the time inverse of the input waveform, $s(t)$. A delay is generally introduced to avoid the problem of non-causal input-output relationships:

$$h(t) = ks(-t + \tau d)$$

Correspondingly in the frequency domain this transforms into:

$$H(\omega) = Ks*(\omega)\ e^{-j\omega\tau}$$

where $s*(\omega)$ is the complex conjugate spectrum. The output of this device becomes by convolution:

$$y(\tau) = k \int_{-\infty}^{\infty} s(t)s(t + \tau - \tau d)dt$$

$$= k\phi_{ss}(\tau - \tau d)$$

and/or

$$Y(\omega) = K\ s(\omega)^2\ e^{-j\omega\tau d}$$

These are the basic equations pertaining to matched filtering where $\phi_{ss}(\xi)$ is the autocorrelation function of the signal $s(t)$. It should be noted that while matched filtering has been shown to effectively maximize signal-to-noise ratios in a white noise environment, it is by itself not a pulse compressive or time-bandwidth product compressive device if the waveform to which it is matched is itself of unity time-bandwidth product, i.e., a rectangular pulse. For any signal to be capable of undergoing pulse compression it must of necessity possess phase dispersion in the frequency domain, or alternatively phase or frequency modulation in the time domain. Phase dispersion is that part of a signal's phase spectrum that is a nonlinear function of frequency. A signal such as a chirp waveform that possesses phase dispersion or phase modulation will by definition have a greater than unity time-bandwidth product. A matched filter will process this waveform in such a manner that will reduce its time-bandwidth product to unity, thereby removing the phase dispersion. This results in an amplification and compression of the input waveform.

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This entire phenomenon can alternatively be viewed as follows: a compressive signal containing phase dispersion is passed through a matched filter. The filter is tailored to this signal in that its phase dispersion is equal to but opposite that of the signal's, thereby cancelling or removing it. This results in no phase irregularities in the processed signal spectrum so that it is altered, i.e., distorted. This form of distortion is desirable in that it is in the form of a narrower pulsewidth with greater peak amplitude.

The usefulness of pulse compression effectively lies in the areas of increased power and resolution capabilities. By resolution, one means how well multiple targets can be resolved in range and/or velocity.

Maximizing a system's detection capability requires maximizing the energy content of the received signal. This can be done simply by amplification or elongation of the transmitted pulse. Unfortunately transmitters are peak power limited and an arbitrary increase in pulse amplitude is not possible. Likewise, increasing pulse duration degrades range resolution capability for extended targets and requires the radar to become blind to close-in targets for a longer period of time. Pulse compression affords a means of transmitting a long (poor range resolution) low amplitude modulated pulse of high time-bandwidth product, receiving it, matched filtering it to obtain an amplified, narrow (good range resolution) pulse. Basic receiver simplicity is lost in this process. System complexity has significantly increased due to delay line sensitivities, weighting networks, gain and phase tolerances throughout the entire processing chain, and possibly multi-channel signal processing.

3. Frequency Modulation

Having shown that some form of modulation is essential to the pulse compression process the question naturally arises as to what types of modulation are suitable or even optimum in some sense. This basic although nebulous area will not be discussed in this report except to state that the system designer must know a priori what the tactical situation is before even beginning to attempt to answer this question. He must be aware of a host of possible pitfalls such as peak power requirements, range and velocity resolution, delay line technology, ECM, clutter environment, sidelobe requirements, target velocities, accelerations, etc.

This report is limited to the investigation of two modulation functions: the linear-FM or chirp and the hyperbolic-FM or chype. The parameter of interest is the waveform's sensitivities to target doppler. It is believed that these modulation functions are significant enough to warrant this special consideration. The chirp modulation is currently the most utilized one available and is indicative of today's

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state-of-the-art in pulse compression radars. The chype modulation, on the other hand, has only recently begun to be investigated based in part on its apparent occurrence in nature, namely in bat and porpoise echolocation. The apparent suitability of chype modulation and non-suitability of chirp modulation with respect to a high velocity target environment is the basis of this report.

4. Doppler Effect

A moving target has the effect of translating the impinging illumination frequencies associated with a signal according to the equation:

$$f_r = \left[\frac{1 + \frac{v}{2}}{1 - \frac{v}{2}} \right] f_t$$

where

f_t = transmitted frequency

f_r = received frequency

v = target velocity

c = velocity of propagation

. = $2v/c$.

The doppler frequency shift, $f_d = f_t - f_r$, becomes

$$f_d = \left[\frac{v}{1 - \frac{v}{2}} \right] f_t \leq v f_t \qquad .$$

It is readily seen that higher frequencies undergo a larger doppler shift than do lower frequencies. This has the effect of dilating or contracting the received signal spectrum as well as contracting or dilating the signal itself. This doppler phenomenon results in a signal whose spectrum is not only shifted, but whose bandwidth has also been transformed:

$$\Delta B = \left[\frac{v}{1 - \frac{v}{2}} \right] B \qquad .$$

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Likewise, its pulsewidth T has been transformed; $\Delta T \triangleq \alpha T$ as a result

of target motion during the illumination interval T. As a consequence, a signal reflected from a closing target will appear at a higher carrier frequency, wider bandwidth, higher PRF, and narrower pulsewidth. In light of matched filtering or pulse compression processing, the signal and filter are now considerably mismatched. The doppler phenomenon effectively imposes additional phase dispersion which the matched filter cannot compensate for. This causes a widening of the resulting compressed pulse as well as a growth in its sidelobe structure. This growth of sidelobe clutter is undesirable since it tends to mask weaker signal returns.

These target effects are relatively undiscernable for many current radar applications. In sonar environments, however, the doppler effect is accentuated and consideration of the problem is warranted especially when using high time-bandwidth waveforms.*

The typical narrowband approach to the effects of doppler is to approximate it by a simple frequency translation. In this report the exact doppler phenomenon is used since it and not its approximation imposes the ultimate limit on amplitude and phase distortions of pulse envelopes and their modulation functions. The narrowband approximation is tantamount to assuming a chirp signal's doppler effected modulation function to be a mere translation with no rotational effect (Figure 1). While this is a valid approximation for low time-bandwidth products (narrowband signals) it does not indicate the waveform's actual limitation imposed by a high enough doppler. Given a transmitted waveform, $s_t(t)$, it has been shown that the doppler received signal, $s_r(t)$, is of the form [2]:

$$s_r(t) = \alpha s_t(\alpha t)$$

where

$$\alpha = \frac{1 + \frac{v}{2}}{1 - \frac{v}{2}} \approx 1 + \cdots$$

The question of how much doppler shift can be tolerated by the modulation function of a pulse compression waveform is discussed next in reference to the chirp and chype waveforms.

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*The velocity of sound in air is approximately 335 m/sec as opposed to 1500 m/sec in sea water. Electromagnetic energy is radiated at approximately 3×10^8 m/sec.

5. Linear-FM (Chirp)

The downswept linear-FM waveform can be expressed as:

$$s(t) = k \cos\left[\omega_c t - \frac{1}{2} \mu t^2\right] \quad 0 \le t \le T$$

where

ω_c = carrier radian frequency

μ = radian sweep rate = $2\pi B/T$

B = bandwidth

T = pulsewidth.

The instantaneous phase variation is quadratic so that its frequency modulation is linear. At any instant of time within the pulse,

$$\omega_i(t) = \frac{d\phi_i(t)}{dt} = \omega_c\left[1 - \frac{\mu t}{\omega_c}\right]$$

The doppler induced distortion of the modulation function causes the received pulse to appear as:

$$s(t) = \alpha k \cos\left[\alpha \omega_c t - \frac{1}{2} \mu \alpha^2 t^2\right]$$

so that

$$\omega_i(t) = \alpha \omega_c\left[1 - \frac{\alpha \mu t}{\omega_c}\right]$$

If an instantaneous period is considered (corresponding to the zero crossings of the modulation function) it is seen to vary in a hyperbolic fashion:

$$T_i = \frac{2\pi}{\omega_i} = \frac{T_c}{\left[1 - \frac{\mu t}{\omega_c}\right]\alpha}$$

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where T_c corresponds to $2\pi/\omega_c$. These relationships are illustrated in Figure 1 where it is apparent that in the presence of doppler, the transmitted and received modulation sweep rates are different and cannot be brought into coincidence or matched with a mere shift in time or frequency. This increase or decrease in modulation slope is a consequence of the doppler center frequency shift and bandwidth transformation. What is evidently optimum in a doppler sense is to have the modulation functions of transmitted and received waveforms equal so that no decorrelation or degradation in the compressed pulse will occur.

The linear-FM modulation function while not posessing the ideal invariance to doppler is in fact tolerant to mild cases of doppler especially if the matched filter bandwidth is slightly larger than required to accommodate small doppler shifts and minimize temporal losses. Its limit of tolerance can be intuitively arrived at by considering a typical return from a closing target. In such a case the pulsewidth is shortened, $T' = T(1 - \nu)$ so that $T' = T - \nu T = T - \Delta T$. If ΔT approaches $1/B$ in magnitude, significant distortions will result from matched filtering. Therefore, $\Delta T < < 1/B$ or $\nu T < < 1/B$ or finally:

$$TB < < 1 \quad .$$

As stated earlier, this implies that for a higher pulse compression effect, slower targets must be illuminated. Thor [3] depicts the effects of utilizing high time-bandwidth product signals in a high velocity target environment and roughly sets the limit of chirp compression ratios at 10,000. This seemingly academic limitation radically changes if a space environment is considered. A case in point is the National Aeronautic and Space Administration's Grand Tour of our solar system's outer planets. Velocities attainable by such payloads approach 25,000 meters/second (55,000 mph) in the vicinity of Saturn. For such radial velocities, linear-FM would be limited to time-bandwidth products on the order of 600 in lieu of a significant increase in receiver complexity to avoid doppler distortions.

6. Hyperbolic-FM (Chype)

The derivation of the hyperbolic-FM modulation function has been arrived at by various authors [4, 5] most notably be equating transmitted and received modulation functions. Hyperbolic-FM can be expressed as:

$$s(t) = k \cos\left[\frac{\omega_c^2}{\dot{\omega}_c} \ln\left(1 + \frac{\dot{\omega}_c t}{\omega_c}\right)\right] \quad 0 \le t \le T \quad .$$

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The instantaneous phase variation is logarithmic so that its frequency modulation is hyperbolic. As Kroszczyński points out [5], this causes the zero crossings to act linearly and thus calls the process linear period modulation:

$$\omega_i(t) = \frac{\omega_c^2}{\omega_c + \mu t}$$

and/or

$$T_i(t) = T_c \left[\frac{\mu t}{\omega_c} + 1 \right] \quad .$$

The doppler induced distortions of these modulation functions can be expressed as:

$$\omega_i(t) = \frac{\alpha \omega_c^2}{\omega_c + \alpha \mu t}$$

and/or

$$T_i(t) = T_c \left(\frac{\mu t}{\omega_c} + \frac{1}{\alpha} \right) \quad .$$

These relationships are depicted in Figure 2. By comparing Figures 1 and 2 an interesting observation can be made: a simple time shift cannot cause coincidence in the chirp case as it can in the chype case. This shift in time is easily calculated from Figure 2 and is expressed as:

$$t_s = \frac{\omega_c}{\mu} \left(\frac{\alpha - 1}{\alpha} \right) \quad .$$

It is interesting to note that for the chirp case the pseudoperiod modulation is hyperbolic. One can simply view this as a reversal in the structure of time and frequency modulations between the two waveforms.

Based upon use of chype modulation it is seen that for an additional delay of t_s seconds during correlation by the matched filter, complete coincidence or correlation will result. This simply means that no degradation of the doppler "degraded" compressed pulse will result. If one returns to the concept of spectral phase dispersion, it is clear that no additional phase dispersion is introduced by the doppler effect that cannot be coped with by a correlation delay. As Rihaczek [6] points out, however, chype modulation does casue ambigious range-velocity measurements since multiple targets at certain key ranges and velocities are rendered ambigious.

In attempting to gain insight into the frequency domain spectral characteristics of the hyperbolic-FM waveform, certain difficulties are encountered. Contrary to the chirp spectrum which is readily expressed in terms of Fresnel integrals (which are well tabulated), the chype spectrum as derived by Kroszczyński [5] takes the following, rather complicated, form:

$$
s(\omega) = \frac{1}{j\omega} \exp\left\{ -j \frac{\omega_c^2}{\mu} \left[\ln\left(\frac{\omega}{\omega_c}\right) - \frac{\omega}{\omega_c} + \ln\left(\frac{2\pi\mu}{\omega_c^2}\right) + \frac{\pi}{4} \right] \right\}
$$

$$
\times \left\{ \gamma\left[1 + j\frac{\omega_c^2}{\mu} \; ; \; j\frac{\omega_c \omega}{\mu}\left(1 + \frac{\mu T}{\omega_c}\right) \right] \right.
$$

$$
\left. - \gamma\left[1 + j\frac{\omega_c^2}{\mu} \; ; \; j\frac{\omega_c \omega}{\mu} \right] \right\} \quad .
$$

In the above expression, γ is the incomplete gamma function.* Sufficient tabulation of this function is nonexistent with respect to complex arguments. As a result, a numerical Fast Fourier Transform (FFT) approach was used for this analysis.

A further interesting resemblence between the chirp and chype modulations becomes evident if the argument of the chype waveform is expanded:

$$
\frac{\omega_c^2}{\mu} \ln\left(1 \mp \frac{\mu t}{\omega_c}\right) = \omega_c t \pm \frac{\mu t^2}{2} + \frac{\mu^2 t^3}{3\,\omega_c} \pm \cdots
$$

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* $\gamma(x, y) = \displaystyle\int_0^y e^{-p} p^{x-1} dp.$

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For low velocity targets and compression ratios the higher terms in the expansion become negligible, resulting in the chirp modulation function. Both the chirp and chype modulation functions result in the same basic pseudo sin x/x compressed pulse structures. While the chirp is bound by the criteria of 'TB · 1, the chype is not. In this sense, chype modulation can be considered doppler invariant whereas the chirp at best can only be considered doppler tolerant.

7. Echolocation

The most interesting assortment of fact relating to acoustic echolocation by bats has only recently been postulated. Based on the tactical situations confronting these mammals, some astonishing parallels between bat echolocation and radar/sonar signal processing becomes evident. Specifically the discussion is limited to Myotis lucifugus and Eptesicus fuscus of the family Vespertilionidae. The ultrasonic* pulses transmitted by these bats are highly modulated and their duration, repetition interval, and bandwidth are adapted to the target environment on a pulse to pulse basis. The search or hunting phase of their transmission is characterized by pulse durations on the order of 5 msec, decreasing to approximately 0.5 msec in the terminal phase prior to intercept. The pulse repetition rate during this same interval typically increases by a factor of 10. Investigations have indicated that the frequency modulation associated with the outgoing sonic pulse takes the form of a nonlinear downward sweep in frequency of approximately one octave. While this octave sweep is a relatively constant feature, it is nevertheless agile in that one pulse may sweep from 80 to 40 kHz while another may sweep from 60 to 30 kHz, depending on the tactical situation. These data indicate time-bandwidth products in the range 20 to 200.

The most intriguing aspect concerning bat echolocation has only recently been postulated. Experiments [7] now indicate that matched filter processing may be taking place in the inferior colliculus section of the brain. This highly developed area of their brain has been shown to possess neuron types ideally suited for target ranging [8]. Apparently their brain stores or memorizes the transmitted ultrasonic signal and correlates it with the received one. A linear frequency sweep or chirp if used by them for echolocation would be greatly limited by doppler in that .'TB · 1. Here .' = 2v/s, where s is the velocity of sound in air (approximately 1.1 ft/msec). Assuming a bat-insect velocity of 15 ft/sec** and a time-bandwidth product of 100, .'TB ⩵ 3. This would

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*The high frequency limitation of the human ear mechanism, generally taken as 20 kHz, render these rather strong ultrasonic pulses (20 to 100 kHz) inaudible.

**Griffin [1] reports velocities for myotis lucifugus in the laboratory on the order of 8 ⌐ . ⌐ 20 ft/sec.

result in severe degradations of the compressed pulse as indicated earlier in this report. When this is considered in light of experimental data indicating a near hyperbolic frequency sweep [9], one cannot help but be cognizant of the doppler invariant nature of this type of modulation when applied to a high velocity insect environment.

The concept of bat pulse compression is reinforced if the spatial resolution capability of the expanded and compressed pulses is considered. A typical 40-kHz sweep over a 2-msec pulse duration results in a spatial path length of approximately 75 cm in air. Such a pulse length is incompatible with the fine resolution requirements of hunting bats. However, if matched filter processing is assumed, the compressed pulse generated in the bat brain as a result of the correlation process then has a processed width of 7.5 mm (75 cm/TB). This coincides with the observed resolution capability of Eptesicus in the laboratory.* Examination of the bat's digestive tracts subsequent to feeding amidst dense insect swarms indicates prey whose wingspans and body dimensions are typically on the order of 5 to 25 mm.

It should perhaps be recalled that the pulse compression technique affords a means of circumventing inherent limitations of signal transmission, regardless of whether it is in a radar's TWT or a bat's larynx. The matched filter, in addition to increasing range resolution capability, maximizes target detectability and signal to noise ratio. This could explain the capability of bats to process extremely faint echo returns (10^{-3} dynes/cm^2) under natural conditions, which up to now has not been satisfactorily explained.

With respect to another possible occurence of chype modulation in nature, Kellogg [10] indicates that the porpoise "whistle" sweeps from approximately 7 to 15 kHz over a 0.5 second interval. This approximate one octave sweep would result in a time-bandwidth product of 3750. In lieu of chype modulation, chirp modulation would be limited to velocities smaller than 0.25 m/sec in sea water. This is hardly compatible with hunting porpoises.

8. Simulation

It was desired to simulate matched filtering of the hyperbolic-FM modulated pulse to ascertain the structure of the resulting compressed pulse for both stationary and high velocity targets. A one-octave frequency sweep was used for the chirp and chype waveforms since this represented a seemingly common occurence in nature. The preliminary

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*It should be noted that Myotis has been observed to avoid 1 mm wire obstacles under controlled laboratory environments.

difficulty of equating swept bandwidths of these two modulation functions was overcome by making the following equality:

$$\eta = \frac{\mu_c}{(\mu_c - \mu_\ell T)}$$

where

η = chype sweep rate

ℓ = chirp sweep rate

T   pulsewidth.

A signal time-bandwidth product of 100 was used (TB · 50 and 200 were also run) such that $0 < TB < 6$. The simulation itself was based on the use of an 11-bit FFT technique. It should be pointed out that while the FFT approach afforded an economical method of computation it was in no way optimized for minimum run time. A typical listing of the program is provided in the appendix. Suitable precautions were taken to minimize various distortions associated with the FFT. This entailed use of a suitable "zero fill" and an analytical signal approach. The zero fill was approximately 80 percent for the forward transforms. No zero fill per se was made for the inverse transform case since this necessity was obviated due to the fact that after spectrum multiplication by the matched filter, data values without the system's passband were already effectively zero. The analytic signal approach was utilized to minimize aliasing distortions associated with the sampling process. The real signal under investigation, s(t), is inputted as:

$$\psi(t) = s(t) + j\hat{s}(t) \quad .$$

Here $\psi(t)$ is the analytic counterpart of s(t), where $\hat{s}(t)$ is the Hilbert transform of s(t):

$$\hat{s}(t) \quad s(t) * \frac{1}{\pi t} \quad .$$

The matched filtering or correlation in the time domain becomes a multiplication of spectral amplitudes and an addition of spectral phases in the frequency domain. In conjunction with a suitable inverse FFT operation the matched filter output is obtained. Figure 3 illustrates the program philosophy.

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9. Results and Conclusions

Figures 4 through 7* illustrate the compressed chype pulse spectra for the zero doppler case. A spectrum ripple effect is present which appears to be inversely proportional to the waveform's time-bandwidth product. This ripple is analogous to the "Fresnel ripple" associated with the compressed chirp pulse spectra, and in the same sense it can therefore be denoted as "gamma ripple." This gamma ripple would cause somewhat inferior compressed pulse performance for very low time-bandwidth products $(T \cdot B \sim 20)$ as in the chirp case.

Figures 8 through 13 clearly depict the high degree of doppler distortion in the compressed chirp pulses as opposed to the almost negligible distortion in the compressed chype pulses for the various target velocities examined. In Figures 12 and 13 there appear to be two target returns in the chirp case as opposed to the one true strong return in the chype case. In this respect chype is invariant to target doppler. The chype modulation is of course not totally "doppler invariant." Temporal overlap losses do occur for high enough doppler causing a corresponding loss in compressed pulse peak amplitude.

Figure 16 shows a comparison of two compressed chype pulses whose time-bandwidth products are 50 and 200. It can be seen that their structure is the same over their mainlobes although there is some slight difference in their sidelobe structure. This is most likely due to the characteristic spectrum ripple being different for the two waveforms, thus causing a slight redistribution of energy over the total pulse duration. Figures 17 through 20 illustrate the total compressed pulse sidelobe structure (existent over T) for varying degrees of doppler. Here, mainlobe width is seen to be invariant (Figures 8 through 13) for the chype modulation. Sidelobe levels do vary as a function of doppler, but they do not increase to any substantial degree.

Figures 21 and 22 are a comparison of chirp and chype modulations $(T \cdot B = 100)$ with respect to maximum compressed pulse amplitude and pulsewidth. The immunity of chype to high velocity target effects is self-evident. Based on these two plots the condition for degraded chirp operation can be taken as approximately $.TB \sim 1$, and in fact this has been the approximation put forward by various authors.

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*Figures 4 through 7 and 17 through 20 are a result of the QUIK3V computer plot subroutines. Figures 8 through 16 are a result of a fourth degree Lagrange interpolation scheme as applied to 50 computer generated data points.
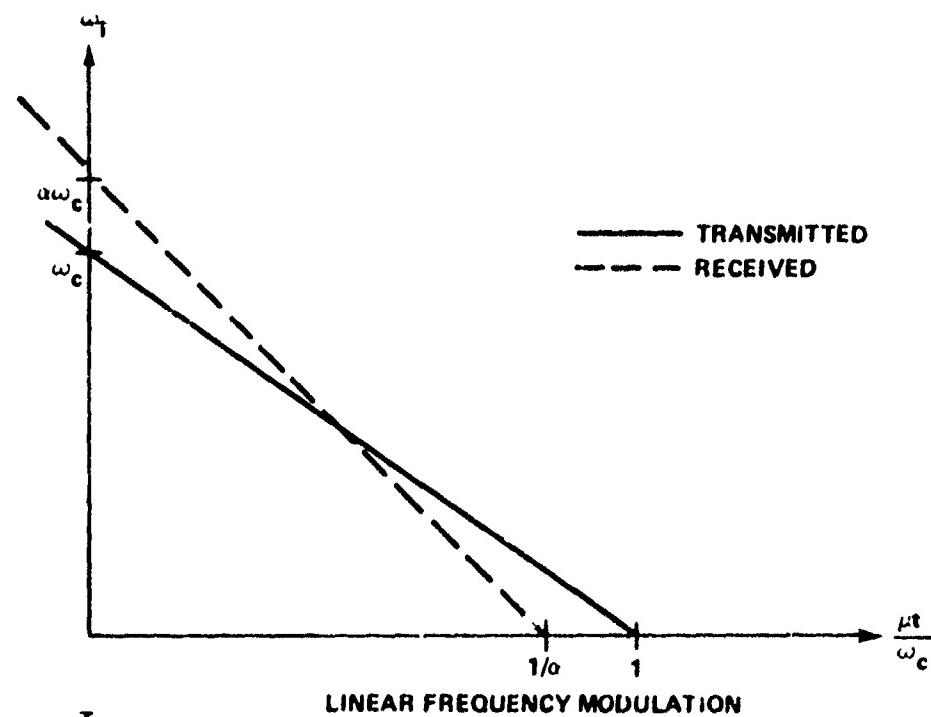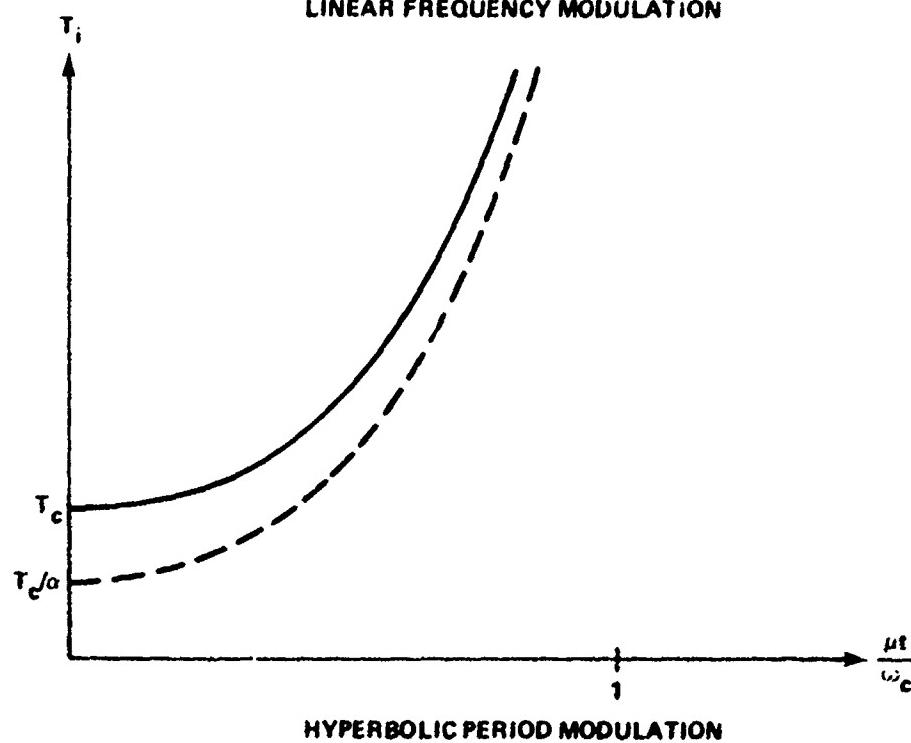
Figure 1. Chirp modulations.

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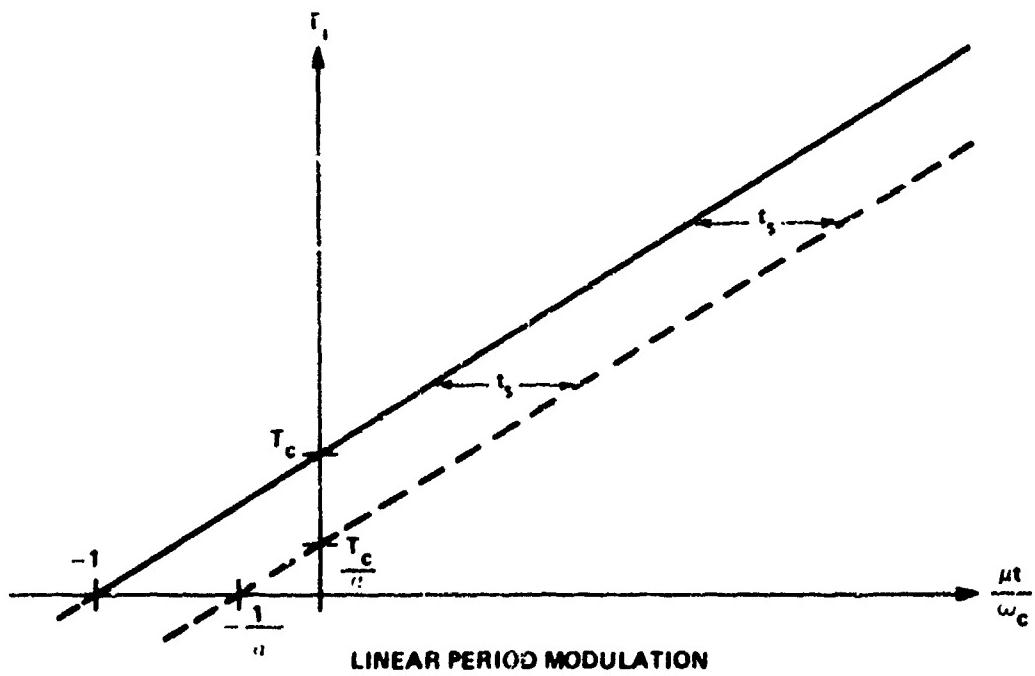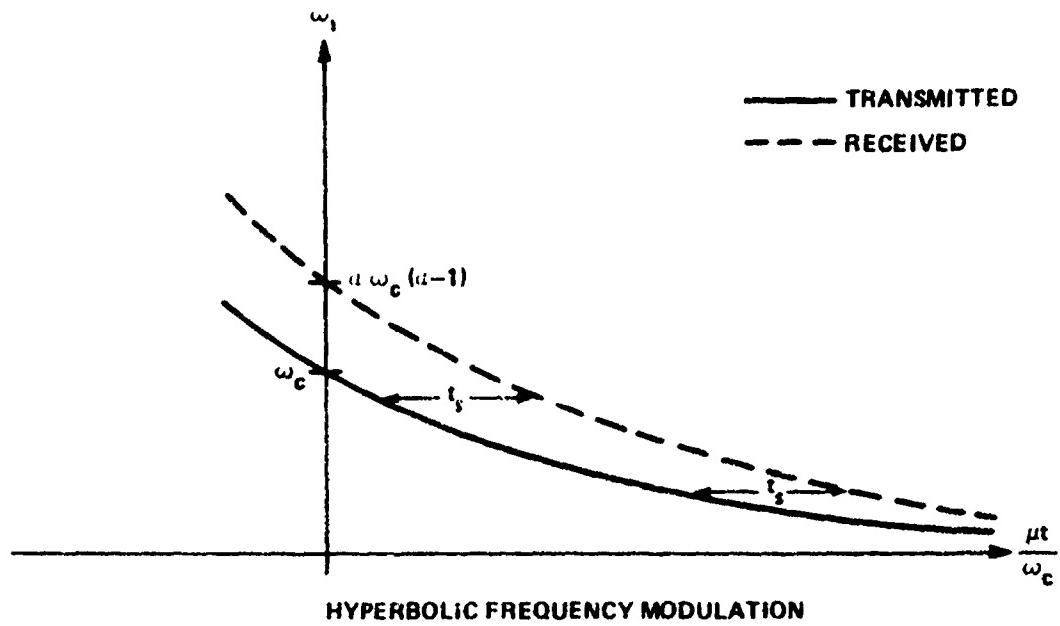
Figure 2. Chype modulations.

17
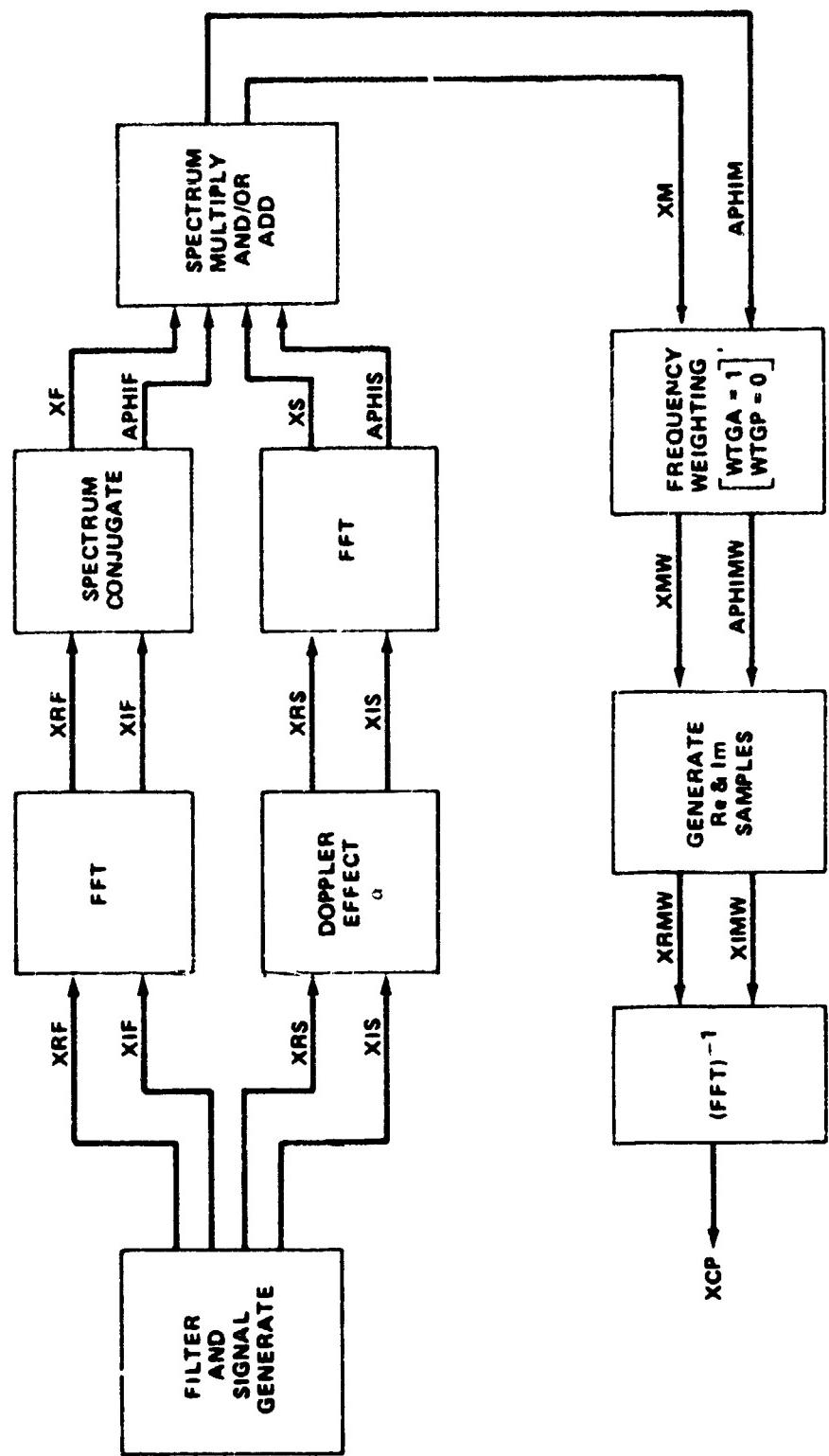
Figure 3. The simulation.

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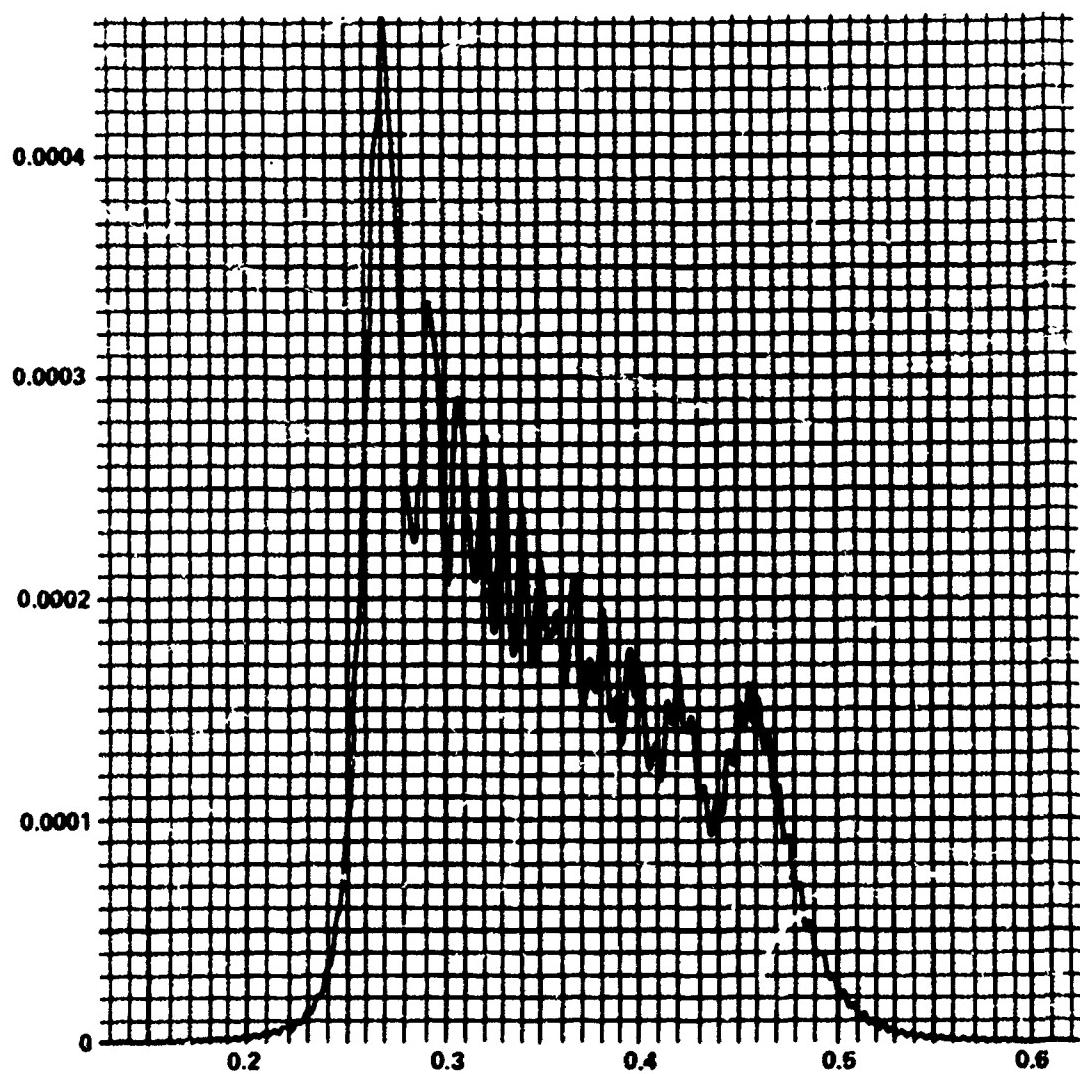
Figure 4. Compressed chype spectrum (T · B = 50, 1 octave).
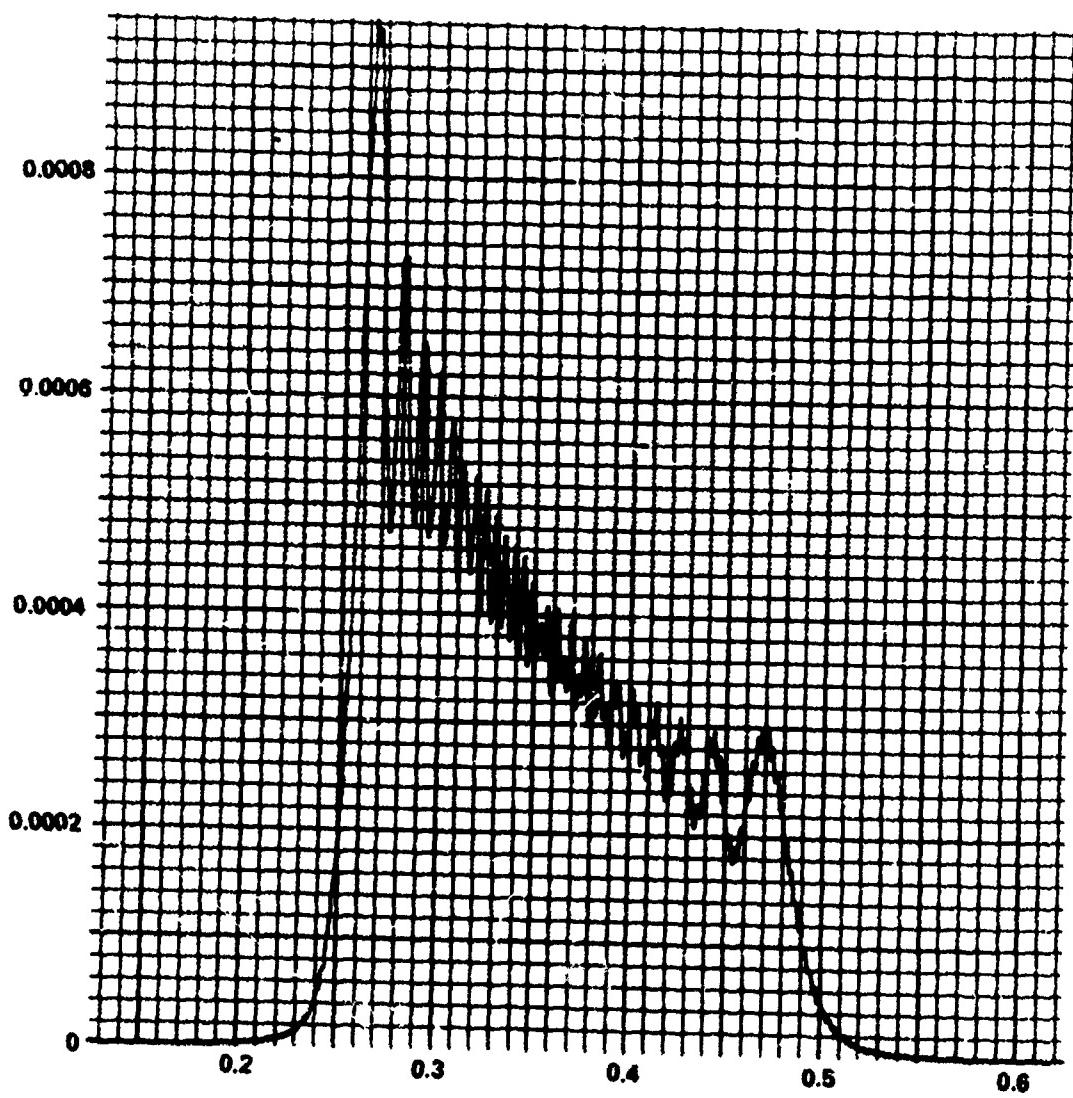
Figure 5. Compressed chype spectrum $(T \cdot B = 100, 1 \text{ octave})$.

20
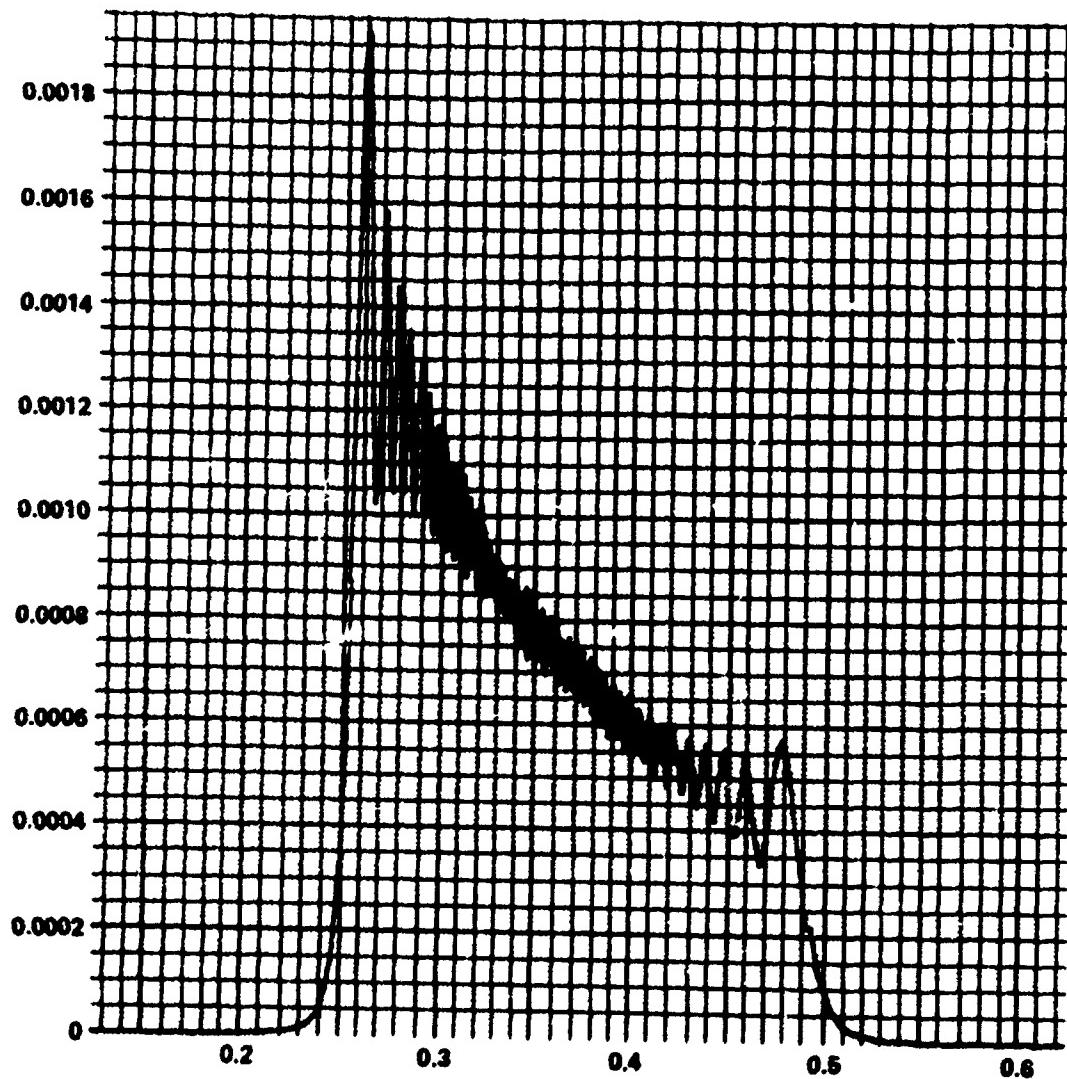
Figure 6. Compressed chype spectrum (T · B = 200, 1 octave).
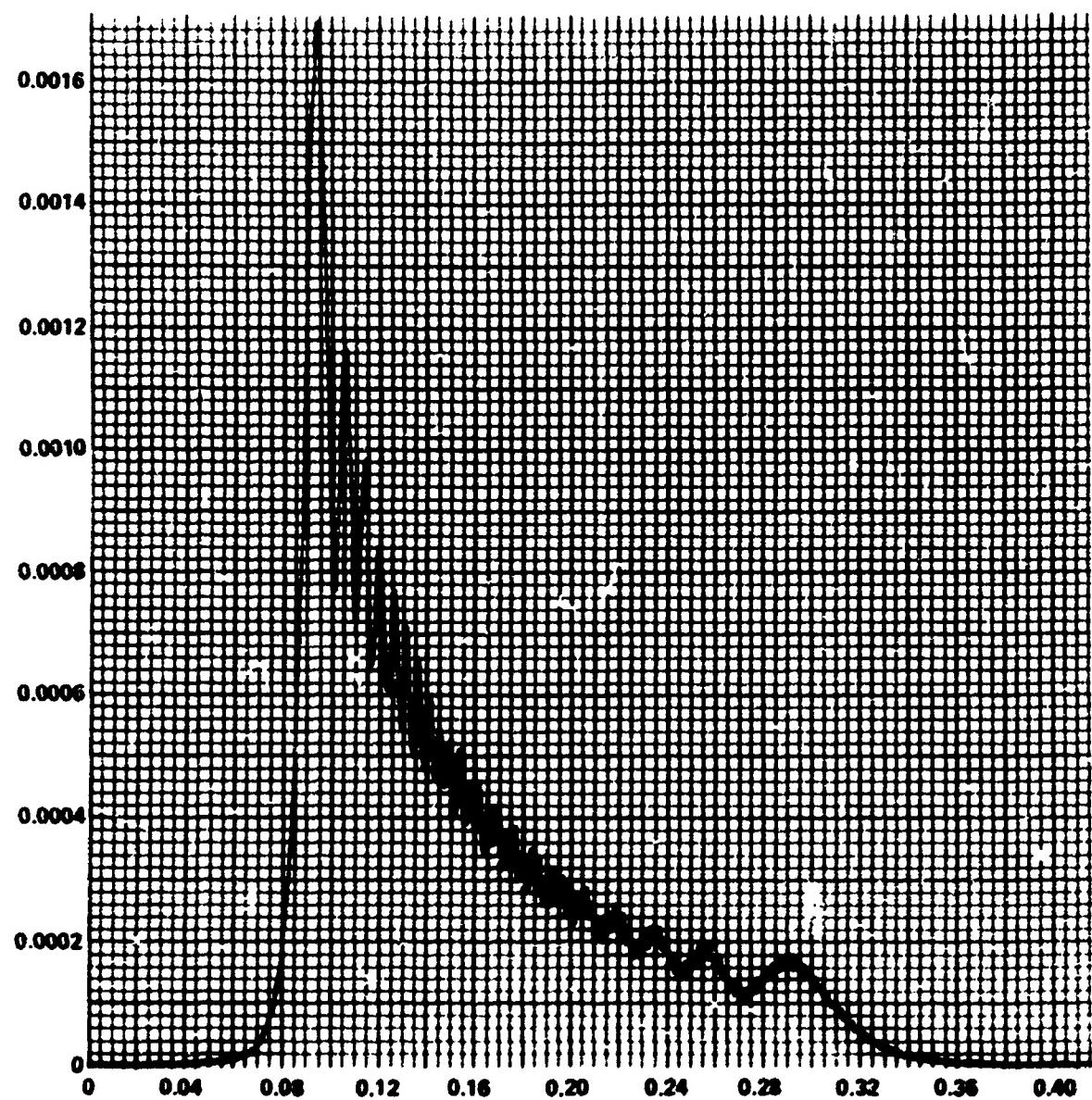
Figure 7. Compressed chype spectrum (T · B = 100, 2 octaves).
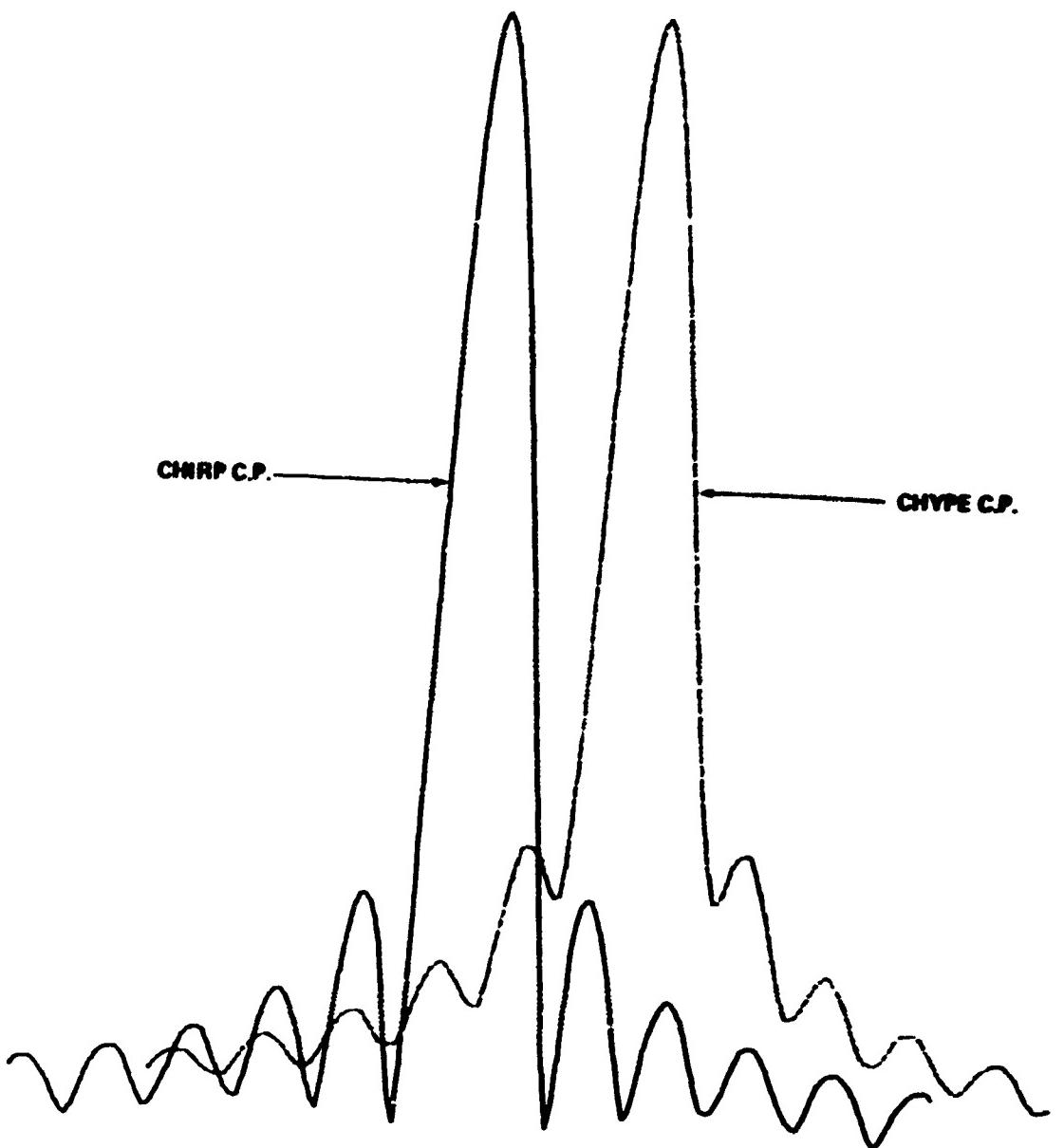
CHIRP C.P.

CHYPE C.P.

Figure 8. Chirp - chype comparison (.TB = 0).
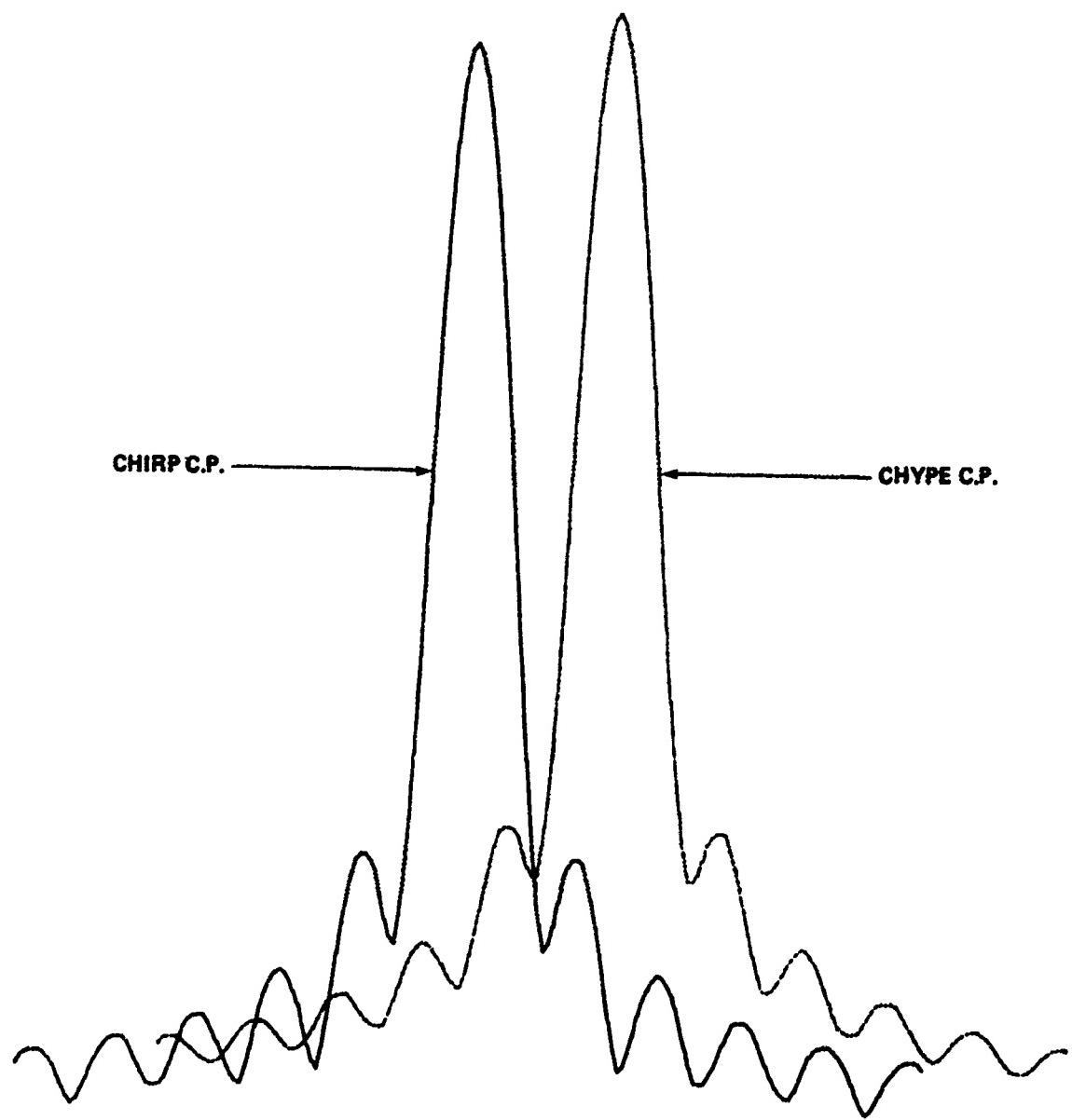
CHIRP C.P. ——→

CHYPE C.P.

Figure 9. Chirp - chype comparison (νTB = 1/2).

Figure 10. Chirp - chype comparison ($\sqrt{TB} = 1$).

25
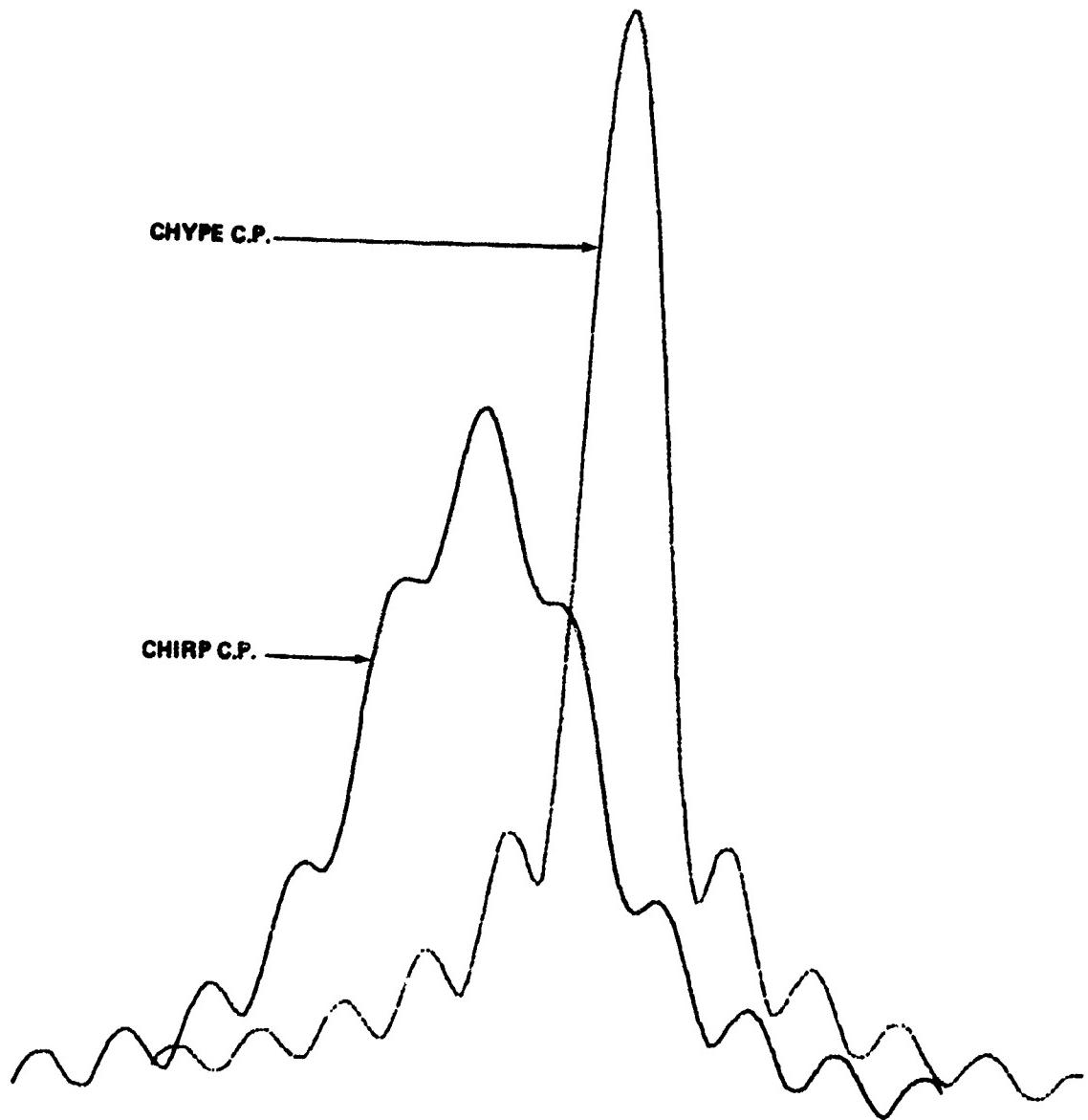
Figure 11. Chirp - chype comparison (TB = 2).

26
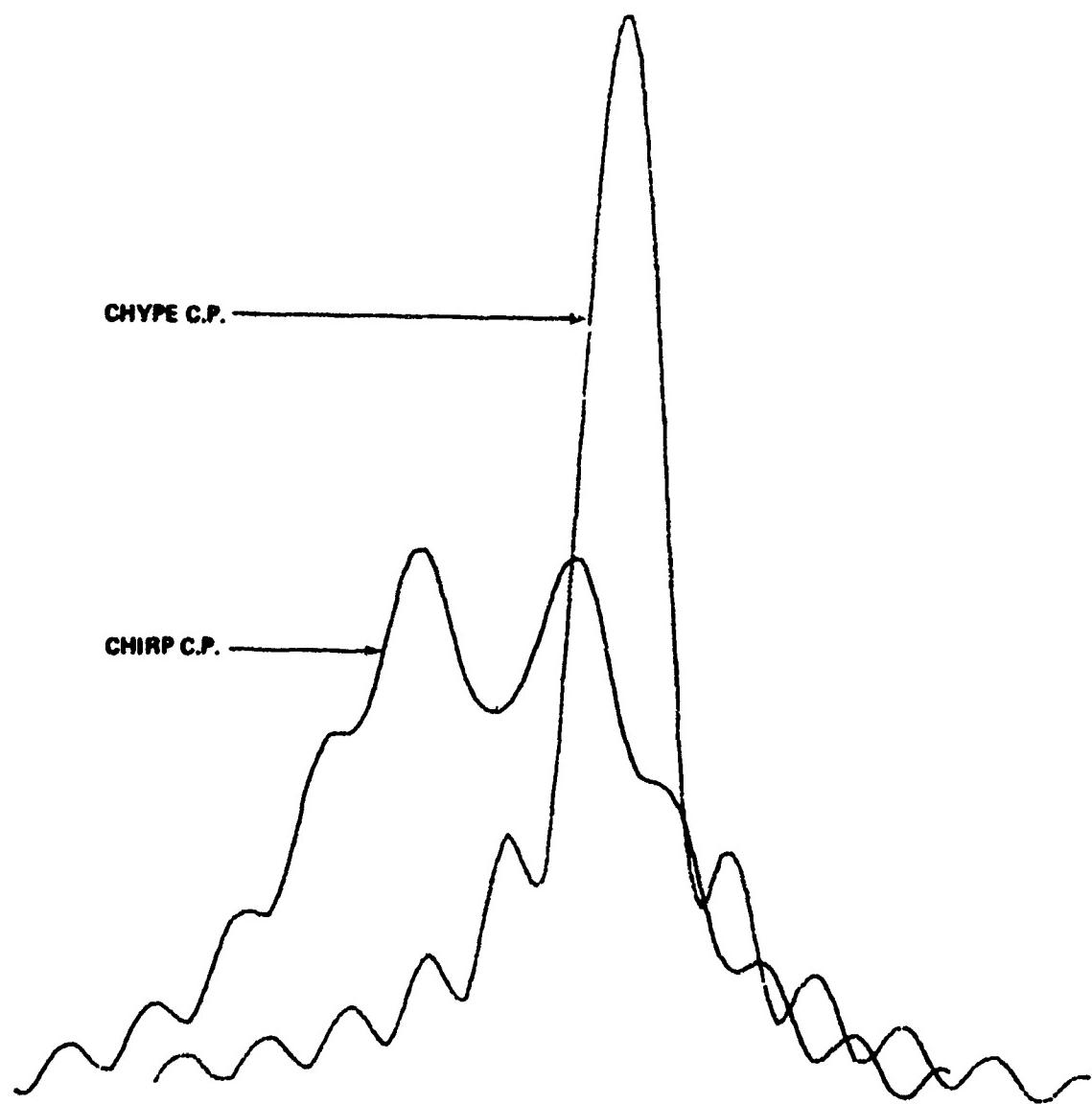
Figure 12. Chirp - chype comparison ($\sqrt{TB} = 3$).
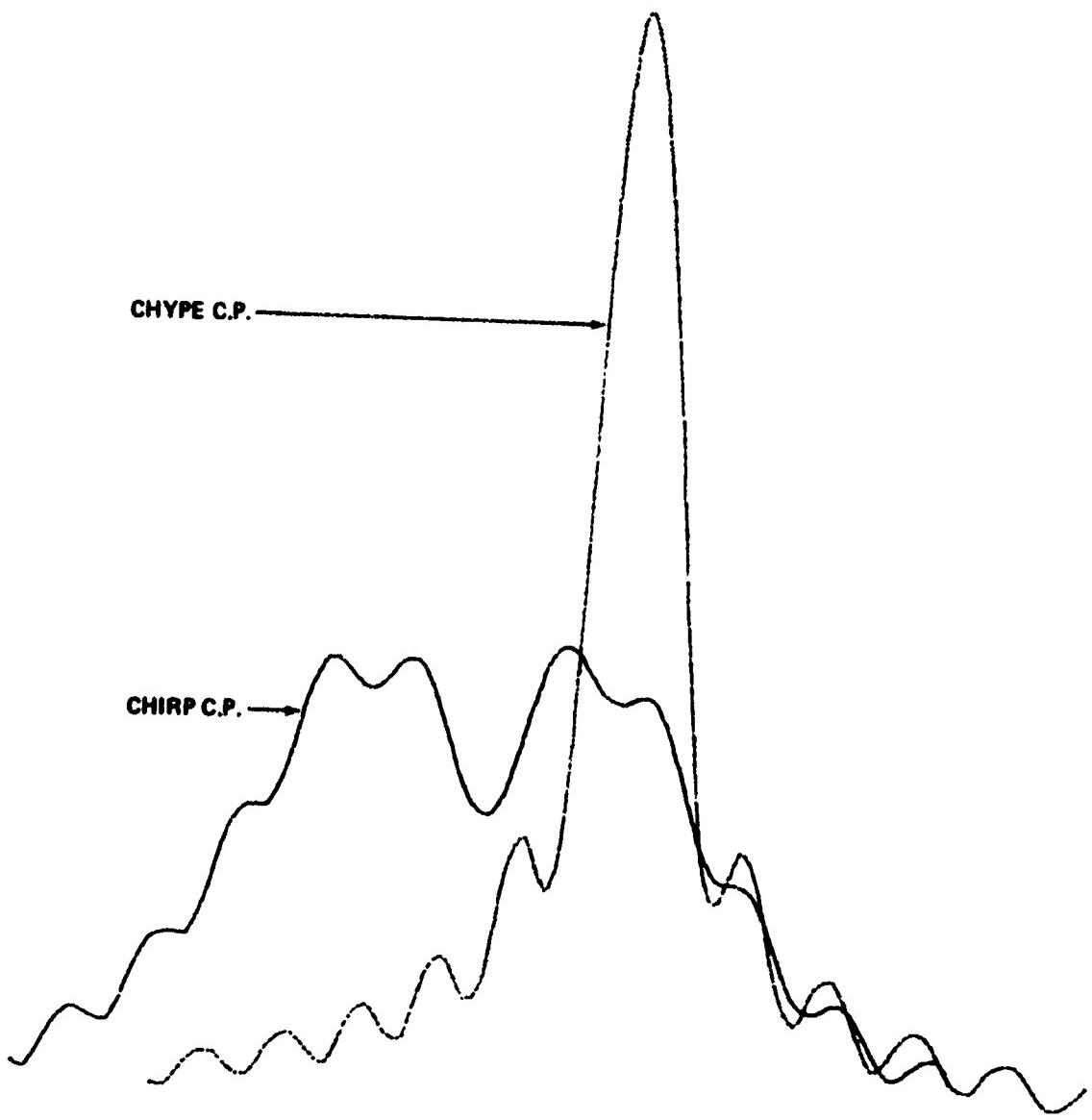
Figure 13. Chirp - chype comparison (.TB = 4).
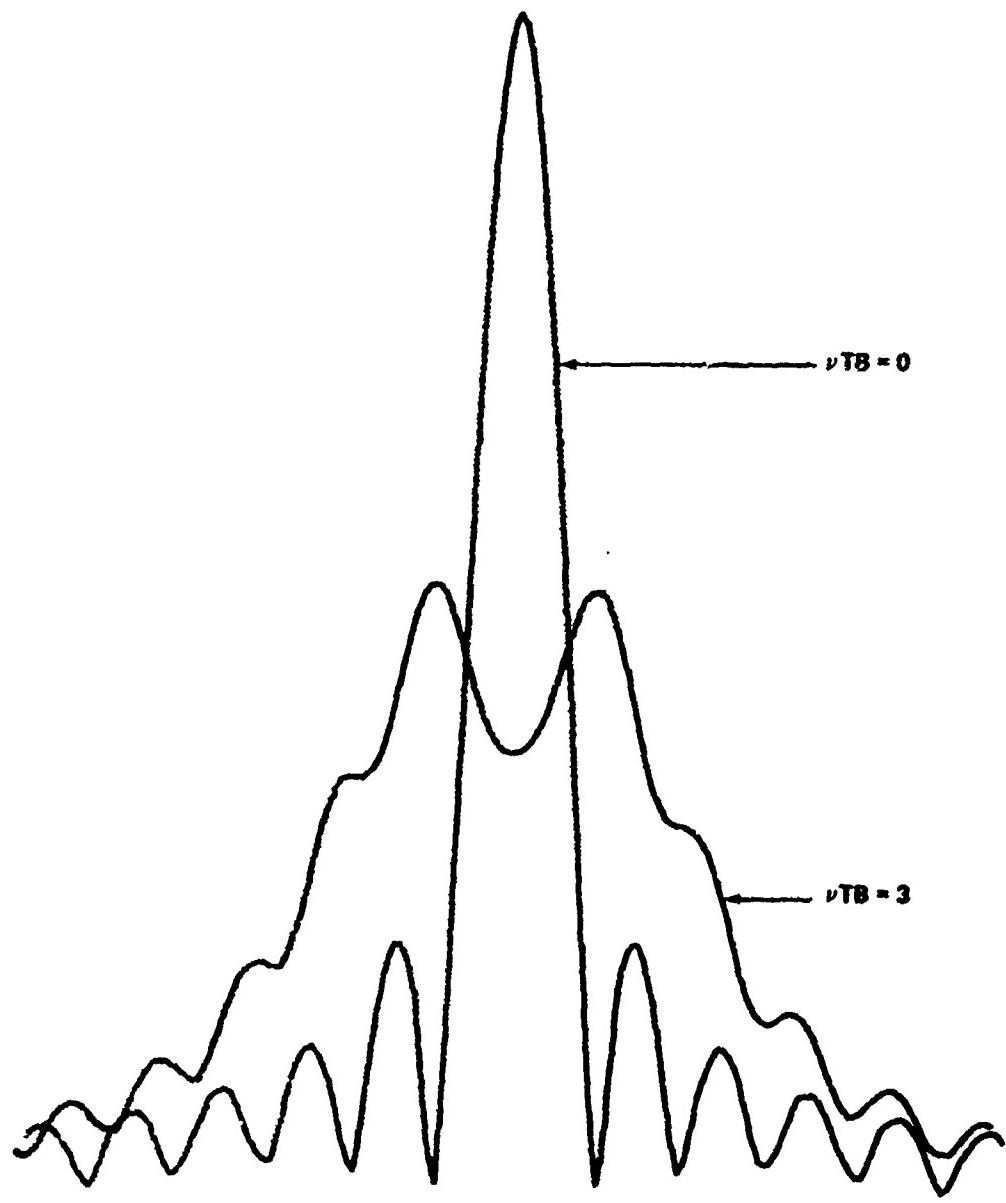
Figure 14. Chirp comparison ($TB = 0, 3$).
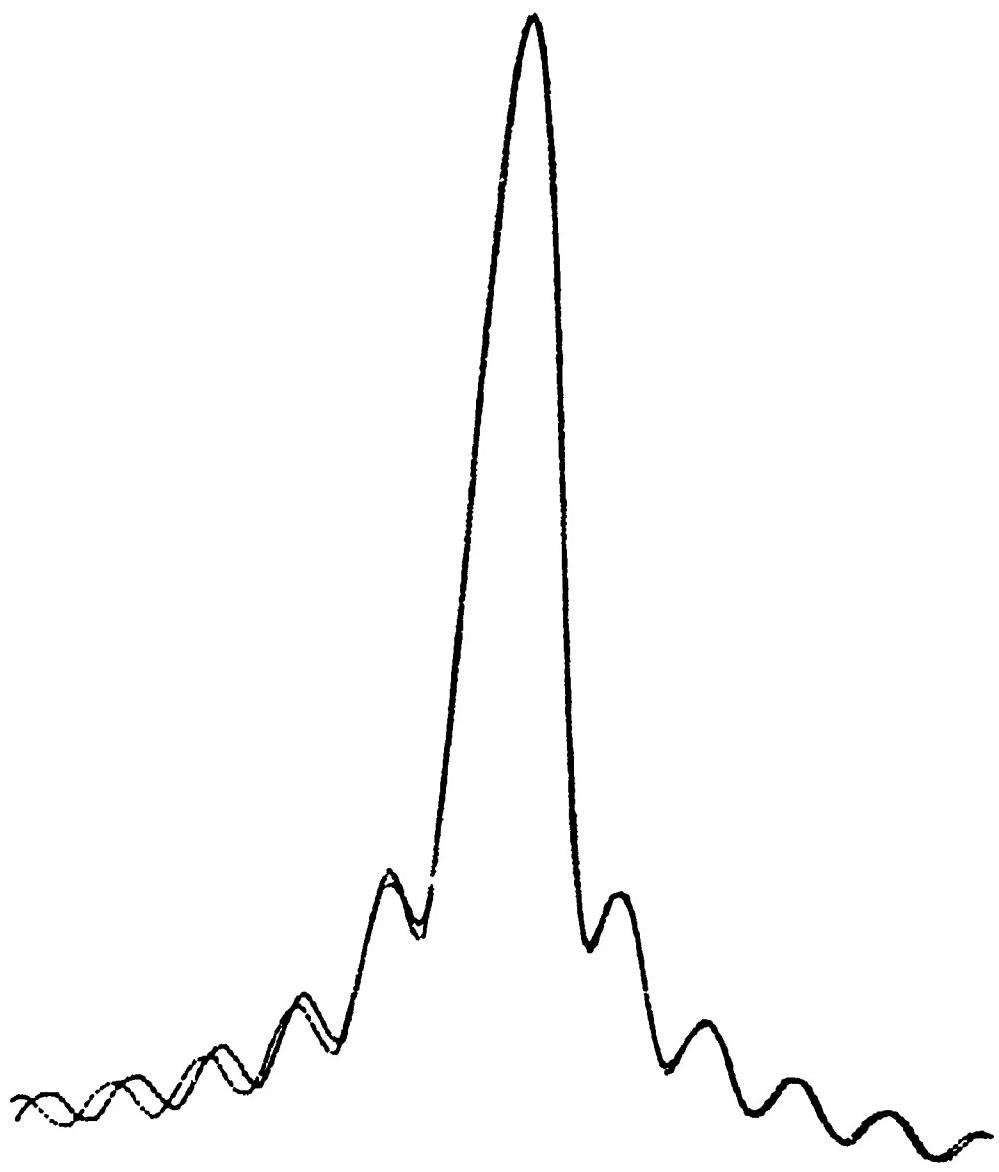
Figure 15. Chype comparison (.TB = 0, 3).
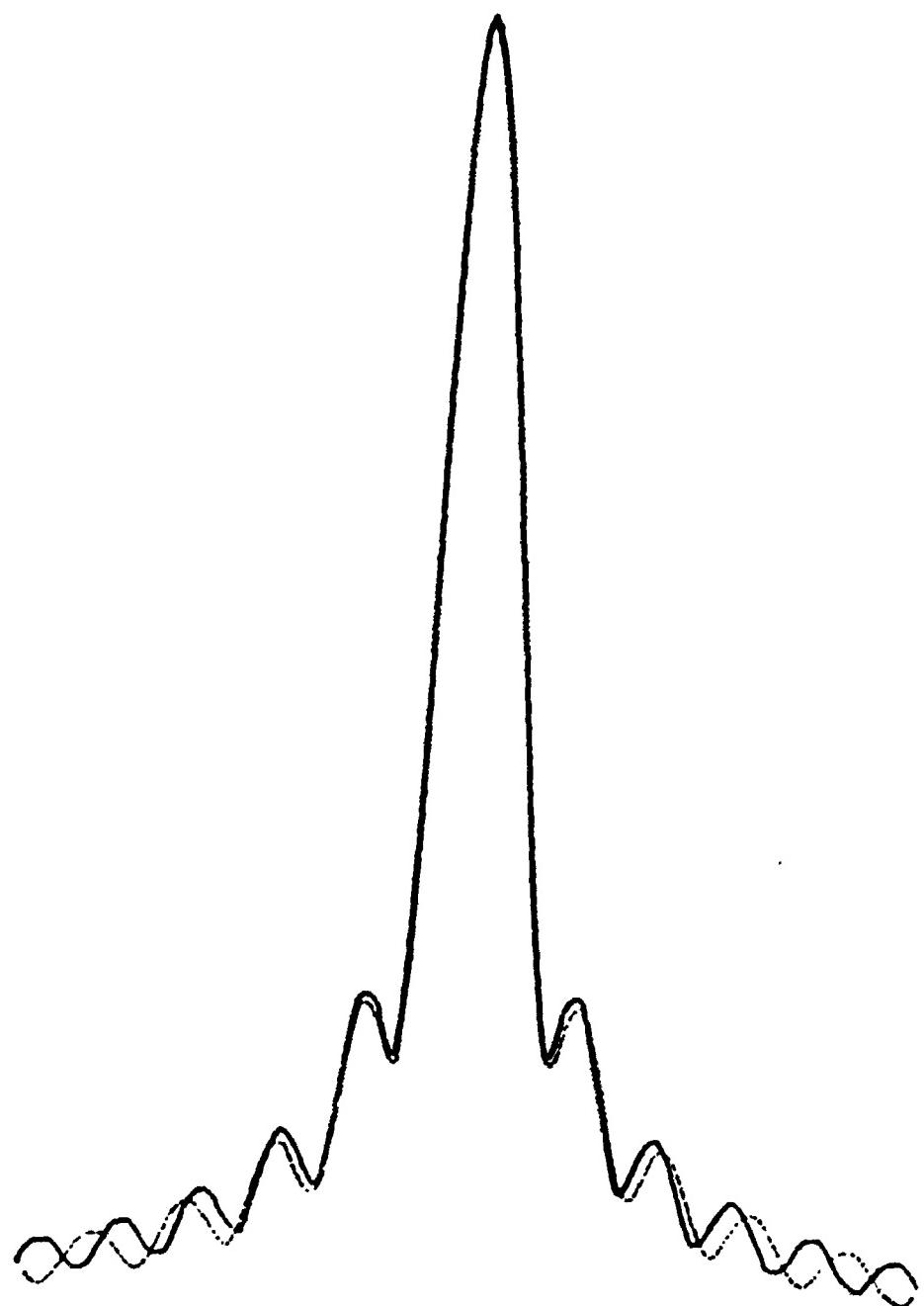
Figure 16. Chype comparison (T · B = 50, 200).
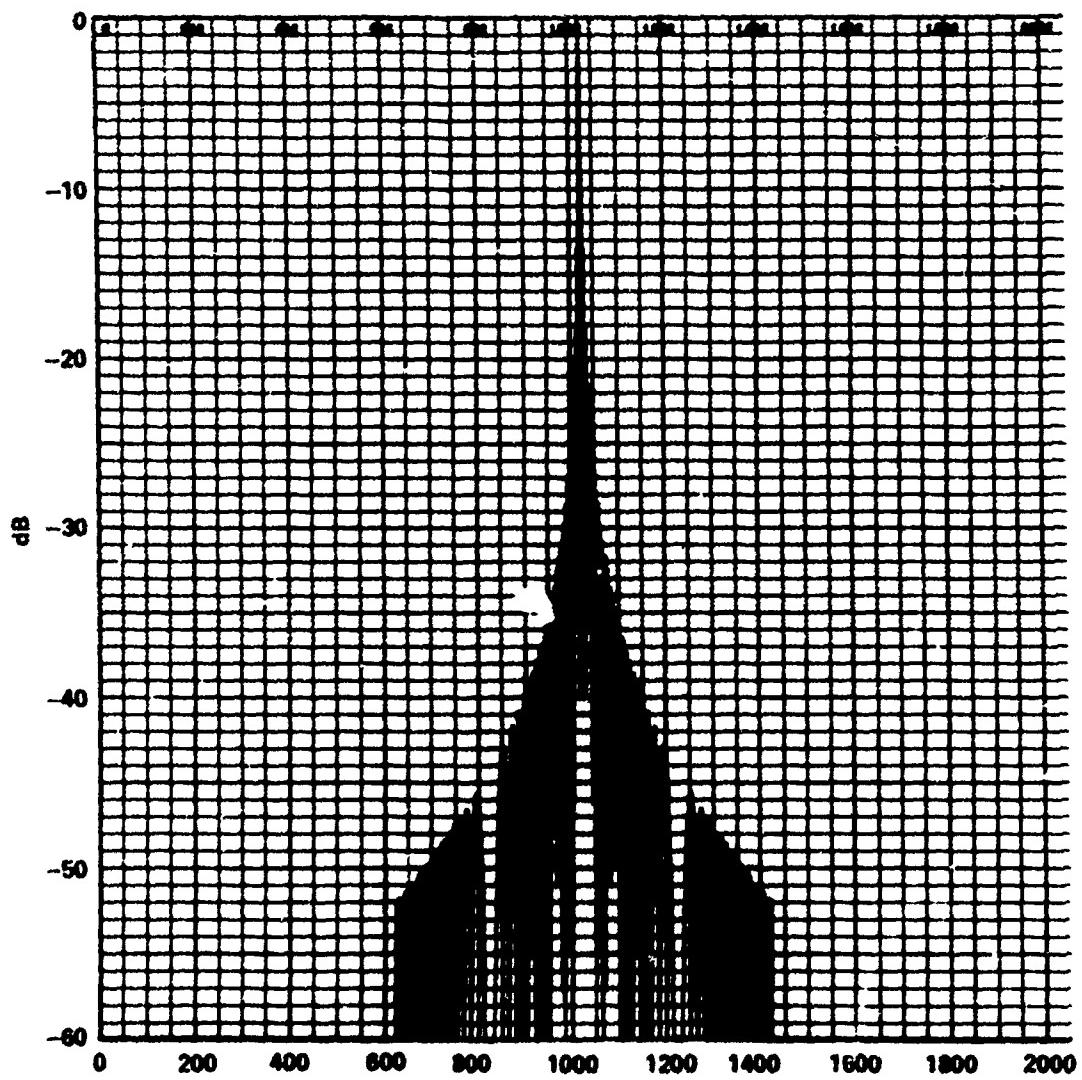
Figure 17. Compressed chirp sidelobe structure (.TB = 0).
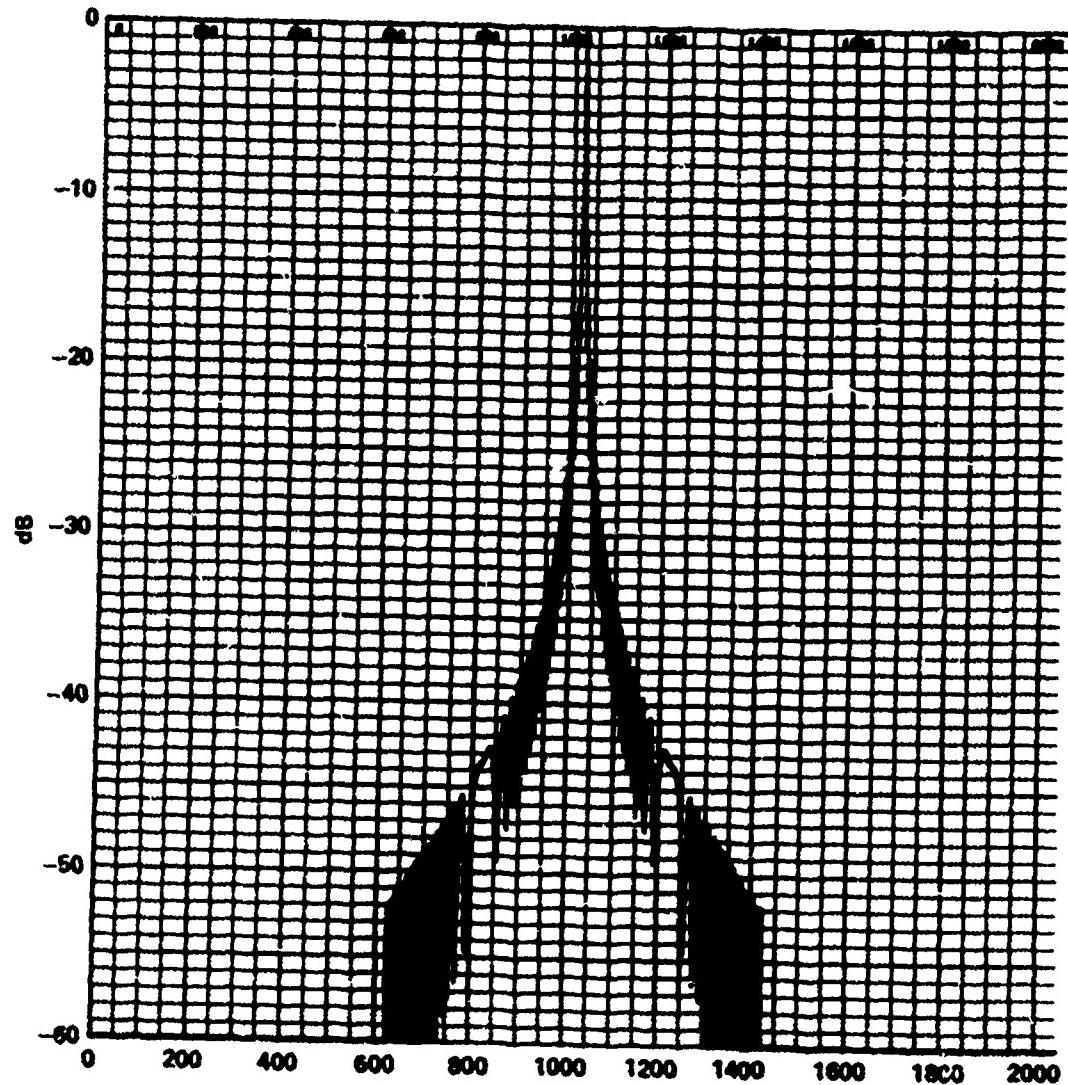
Figure 18. Compressed chype sidelobe structure (TB = 0).

33
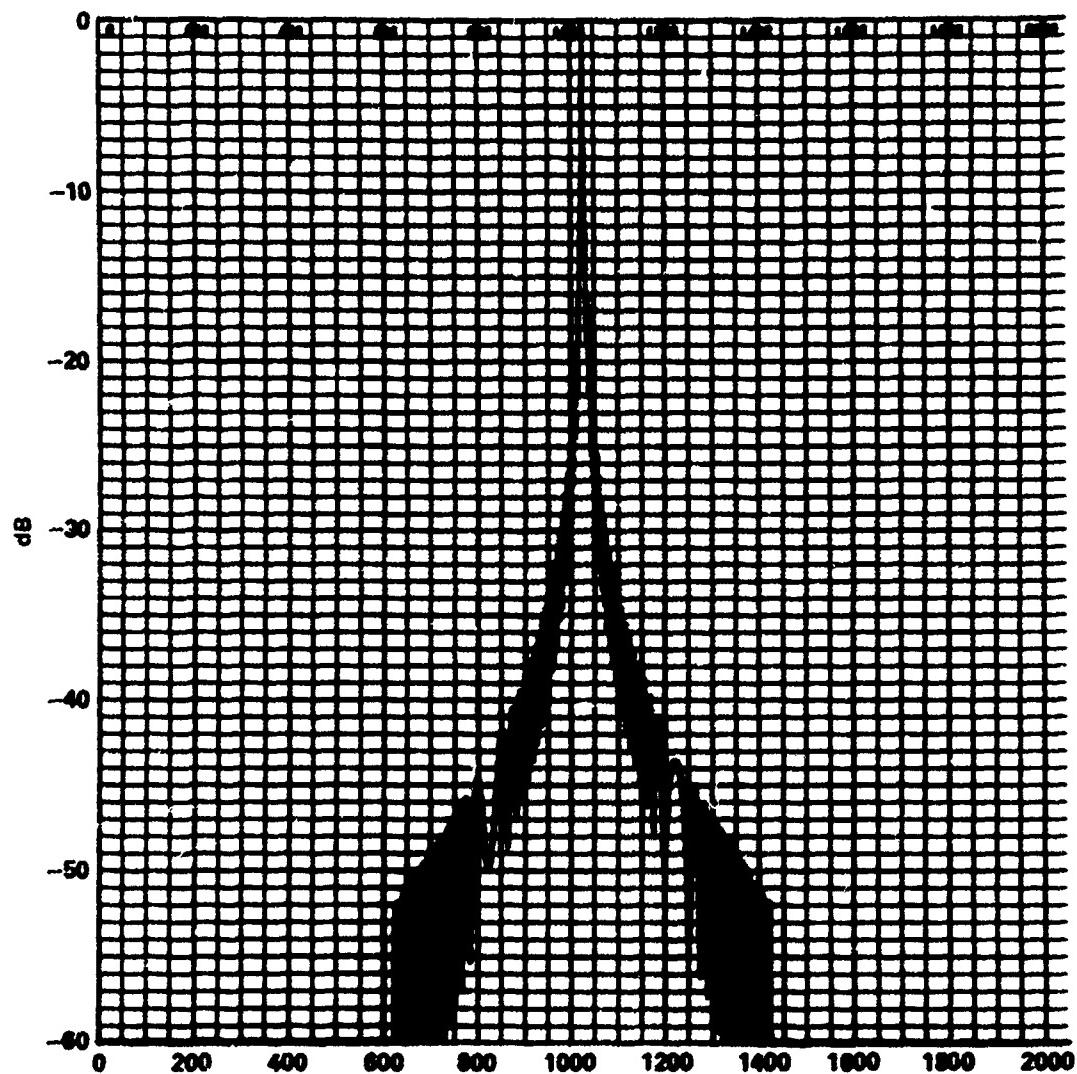
Figure 19. Compressed chype sidelobe structure ($.TB = 1/2$).

34
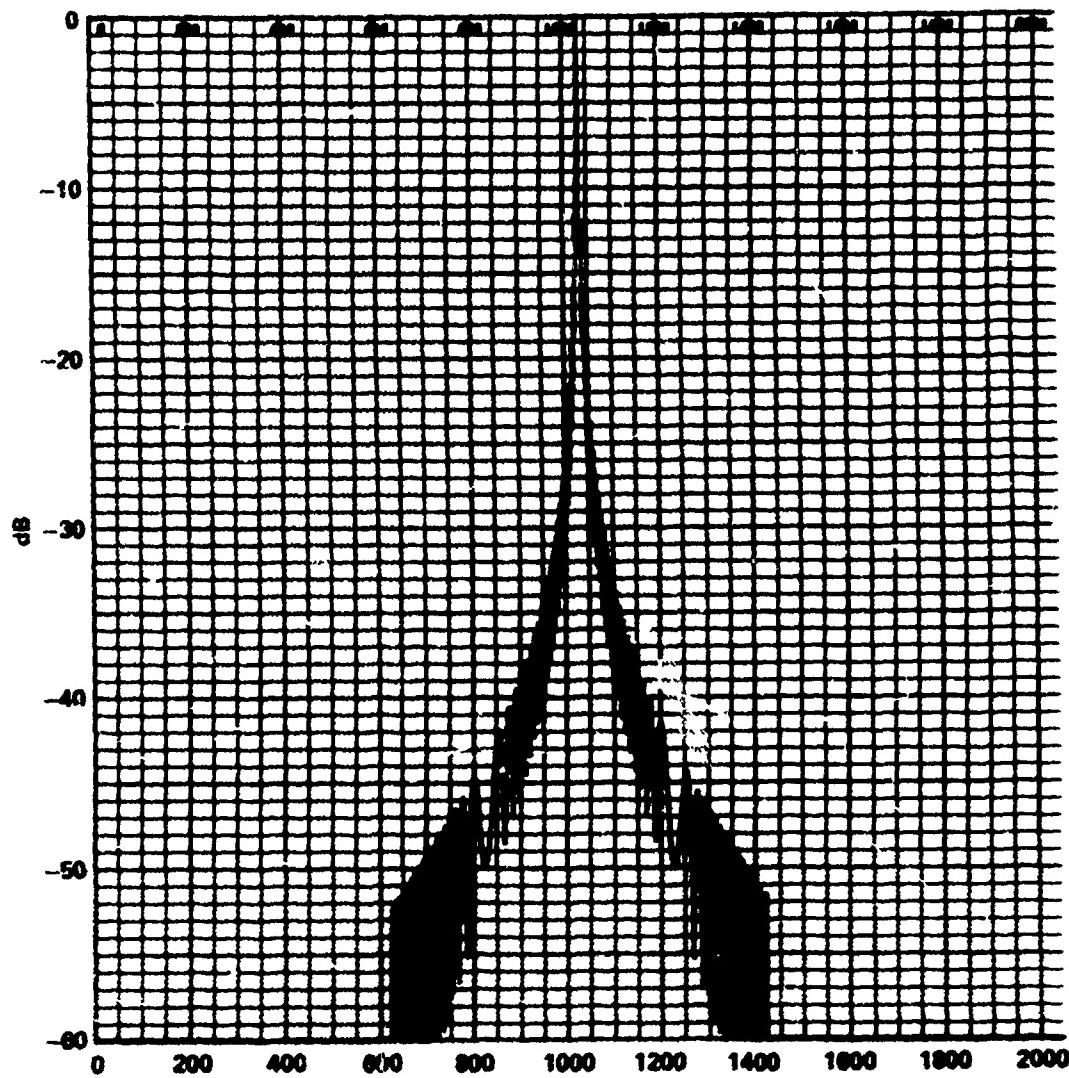
Figure 20. Compressed chyrp sidelobe structure (TB = 2).

35
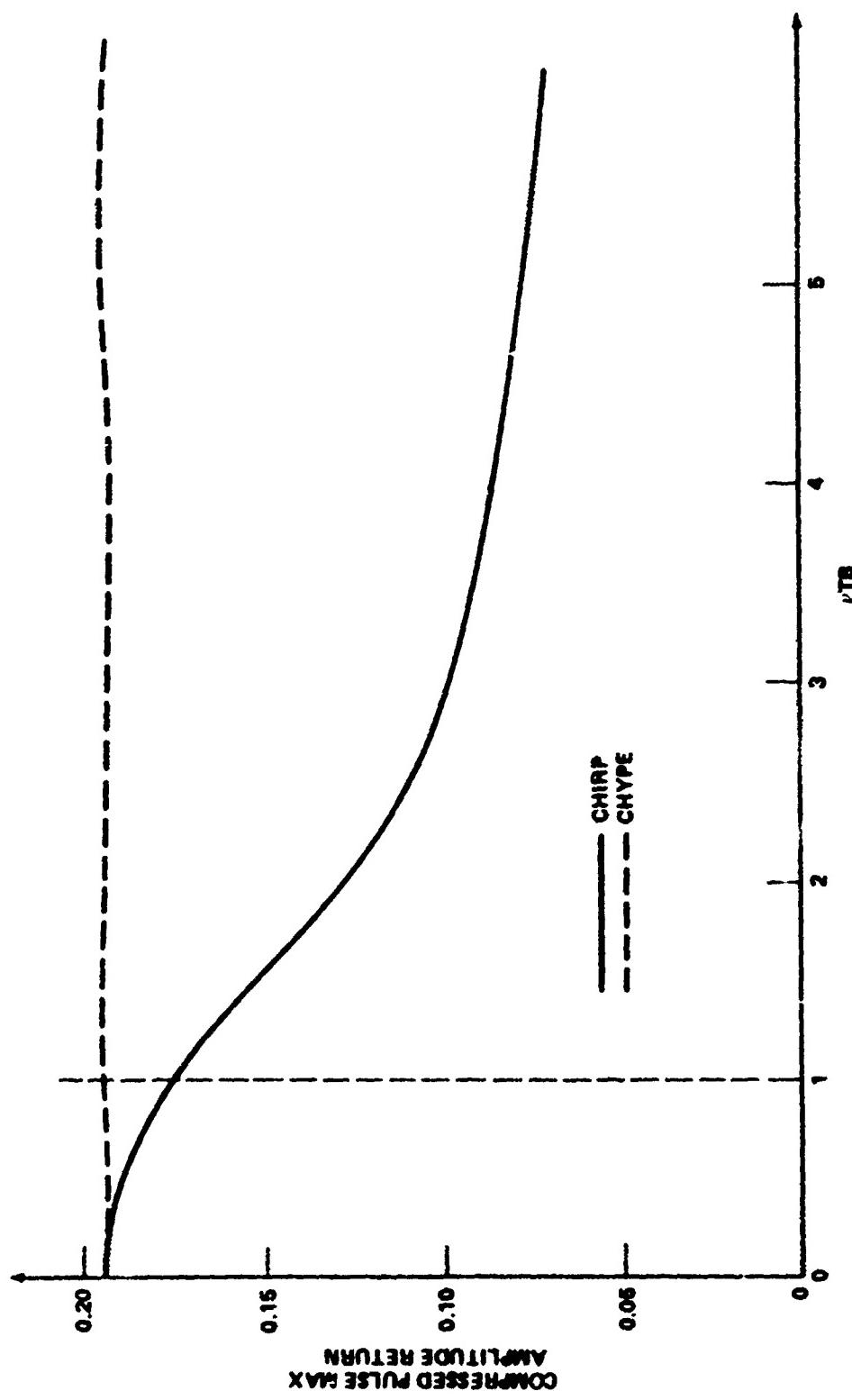
Figure 21. Compressed pulse maximum amplitudes ($T \cdot B = 100$, 1 octave).

36
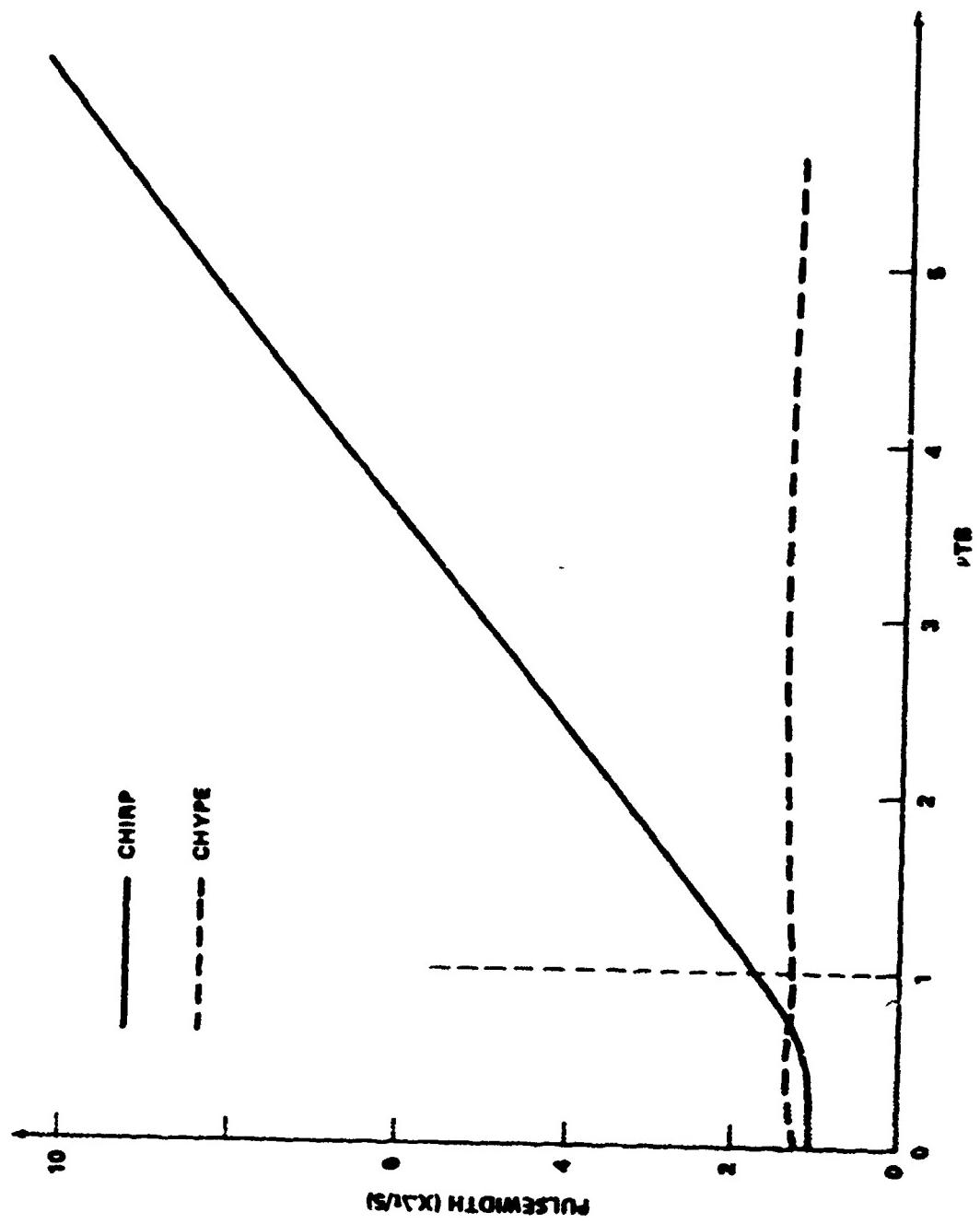
Figure 22. Compressed pulsewidths (T · B = 100, 1 octave).

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Appendix A. TYPICAL PROGRAM USED IN SIMULATION

Included in Appendix A is a typical program used in this simulation.
With the aid of Figure 3 the listing is self-explanatory.

```
      PROGRAM (INPUT,OUTPUT,TAPE6=OUTPUT,INPUT,TAPE5=INPUT,FILMPL=385)
      DIMENSION XR=S(2048),XTS(2048),XRF(2048),XIF(2048)
      DIMENSION XS(2048),XF(2048),APHIS(2048),APHIF(2048)
      DIMENSION FLOX(12),FLOY(12)
      DIMENSION XM(2048),APHIM(2048),WTGA(2048),WTGP(2048),XMW(2048)
      DIMENSION APHIMW(2048),XRMW(2048),XIMW(2048),FREQ(2048)
      DIMENSION TIME(2048),AGP(2048)
      EQUIVALENCE (XRS(1),XP(1)),(XIST(1),APHIM(1)),(XRF(1),WTGA(1)),
     1 (XIF(1),XIMW(1)),(WTGP(1),XMW(1)),(APHIF(1),APHIMW(1)),(XS(1),
     2 XP(1)),(XF(1),XTS(1)),(XM(1),FREQ(1)),(APHIM(1),TIME(1)),(WT
     3 GP(1),AGP(1))
      READ(5,7) FLOX,FLOY
    1 FORMAT(1H ,12,2E12,4/)
    3 FORMAT(///////////55X,31HCOMPRESSED PULSE ENVELOPE IN DB/)
    5 FORMAT(45X,91(******************************************///))
    7 FORMAT(E12,4,2E12,4)
    8 FORMAT(////////65X,32HCOMPRESSED PULSE ENVELOPE NO DB*/)
    9 FORMAT(40X,(12H**************************************////))
   11 FORMAT(//////////40X,35HCOMPRESSED PULSE AMPLITUDE SPECTRUM/)
   13 FORMAT(4X,1H,1H**************************************////)
   16 AGP WRITE(//////////////52X,9HX SPECTRUM)
      IPI=I.14
      I=2(1+1)
      K=2048(1+1)
      DX=(1+X-XL)*N*(L)
      XPL=FREQ(1)*(DELT)
      N=XP*N*(1)*T*
      T=DELTA*N*(1,K)
      TT=2**(1,N=1,SPEC)+12/
      T=FLOAT(N)
      FC=N,2,K
      FCF=FS+N,125
      WT1,N=1
C     TRANSFORM
LI    CONTINUE
      PI=3,1415926536
      IM=2,45
      SUM=SUM,RETURN
      SUM=ZERO
      IFL=1,
      N,IF,X=XS(L)+XS(K)
      XLP=N*I,N+Y
      XI,X=Z1
      SUM=Z,RETURN
      XT PH=(I+X+XS=N()//(TIME*DX=AMDV*)
      XY=XS*(I+X+1=(REPEAT=REPEAT)+112
      N=1+1
      N=N,ROT*
      WLF=1,/I,
      RETURN
      END
C**********************************SIGNAL***************************************
      DO 1 J=1,K,1
      IF((J,1,1),OR,(J,EG,K))CONST=0,5
      IF((J,1,1),OR,(J,LT,K))CONST=1,0
```

```
      ...(X...)...
      ...

      ...(...3*2*PI)/...
      ...     ...(1.+(...JS=1))/...(AC)
      XTS(JS)=...
      XTS(JS)=...
   ... CONTINUE
C*************************** SIGNAL  a )V*************************************
      ...  FFT(X=S+XTS*L)
      ...
      YS(JJ)=SQRT(XTS(JJ)**2+ZTS(JJ)**2)
      IF ((X.S(JJ)...AND...(XTS(JJ).GT...)) APHTS(JJ)=ATAN(XTS(JJ)/
     1XS(JJ))
      IF((XTS(JJ).LT...)..AND..(XTS(JJ).GT...)) APHTS(JJ)=PI+ATAN(XTS(J
     1J)/XS(JJ))
      IF((XTS(JJ).LT...)..AND..(XTS(JJ).LT...)) APHTS(JJ)=PI+ATAN(XTS(J
     1J)/XS(JJ))
      IF((XTS(JJ)..GT...)..AND..(XTS(JJ).LT...)) APHTS(JJ)=ATAN(XTS(JJ)/
     1XS(JJ))
      IF((XTS(JJ).GT..0.)..AND..(XTS(JJ).EQ...)) APHTS(JJ)=0.0
      IF((XTS(JJ).LT.0.).AND..(XTS(JJ).EQ.0.)) APHTS(JJ)=PI
      IF((XTS(JJ)..EQ...)..AND..(XTS(JJ).GT.0.)) APHTS(JJ)=PI/2.
      IF((XTS(JJ).EQ.0.).AND..(XTS(JJ).LT.0.)) APHTS(JJ)=-PI/2.
      IF((XTS(JJ)..EQ...)..AND..(XTS(JJ).EQ.0.)) APHTS(JJ)=0.0
   39 CONTINUE
      CALL  ...SH (APHTS)
C****************** FILTER FUNCTION RFL0************************************
      DO ...  KS=1.PT4
      IF((KS.E..1).OR..(KS.EQ.M)) XREAL=0.5
      IF((KS..1.1)..R..(KS.L.1.M)) XREAL=1.0
      IF(KS.GT..M) XREAL=0.0
      ARG1=(...*6*(KS-1))/XREAL
      ARG2=ALOG(9.+(...*RFLDAT(KS-1))/ORFGAC)
      XRF(KS)=X*ALP.LS(ARG1*ARG2)
      ZRF(KS)=X-ALP.SIN(ARG1*ARG2)
   .. CONTINUE
C****************** FILTER FUNCTION AQUF************************************
      ...  FFT(XRF+XRF+L)
      ...
      ...
      YRF...=SQRT((XRF(KK))**2+(ZRF(KK))**2)
      IF((XRF(KK)..GT...)..AND..(XRF(KK).GT.0..)) APHRF(KK)=ATAN(XRF(KK)/
     1XRF(KK))
      IF((XRF(KK).LT.0..)..AND..(XRF(KK).GT.0..)) APHRF(KK)=PI+ATAN(XRF(K
     1K)/XRF(KK))
      IF((XRF(KK).LT.0.)..AT.0.)..(XRF(KK).LT.0..)) APHRF(KK)=PI+ATAN(XRF(K
     1K)/XRF(KK))
      IF((XRF(KK)..GT.0.0.)..AND..(XRF(KK).LT.0.0.)) APHRF(KK)=ATAN(XRF(KK)/
     1XRF(KK))
      IF((XRF(KK)..GT.0.0.)..AND..(XRF(KK).EQ.0.0.)) APHRF(KK)=0.0
      IF((XRF(KK).LT.0.0.)..AND..(XRF(KK).EQ.0.0.)) APHRF(KK)=PI
      IF((XRF(KK)..EQ.0.0.)..AND..(XRF(KK).GT.0.0.)) APHRF(KK)=PI/2.
      IF((XRF(KK)..EQ.0.0.)..AND..(XRF(KK).LT.0.0.)) APHRF(KK)=-PI/2.
      IF((XRF(KK)..EQ.0.0.)..AND..(XRF(KK).EQ.0.0.)) APHRF(KK)=0.0
```

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```
   CONTINUE
      C???=AY???(J???)
      R  ?R JJ?1)=?A?
      Y??(JJ)=XS(JJ)*XF(JJ)
      ???I/(JJ)=A-?LS(JJ)+?P??F(JJ)
   ?? CONTINUE
      CALL AP?ASE(??H=?X)
      ???T? JK=MIN,MAX
      ?T??(JK)=1.?
      ?TA?(JK)=?.?
   ?? CONTINUE
C  ?? ?IG?T?F?
C ?? ?U??OSES ?F THIS SIMULATION IT IS ASSUMED THAT THE ?L?G?P?NG
C  ??OCESS I? ?I?STA?TE?EOUS, T.?.. THERE IS ?O INHERENT TIME DELAY
C  (L?AR? ?A??E?) ?SSOCIATED ?I? THE NET?O?I?G ?RO?CESS.
      ?? ?? I JK=?K,?4?K
      ??X?(??I)=A?(?I)?X?IN?A(?J)
      ?P?I?.(I??)=??I?(?I?)+?T?X(?I?)
   4? ?O?TI?U?
      CALL ?U??SE(A?H??P?)
      ??ITE(?,?)
      ???TE(?,?I)
      ??RE?(??I? ?A??(?K)? ??5?I=??M?X)
   7? ?? KK=??,?A?
      X??A?(?K)=X?(?K?+LTS(?O?)*?(?K)?
      X???(KK)=X?S(KK)*SI?(?O?I?.?(?K?))
   ?? ?O?TI?U?
      ?? 5? I??J??? ?AX
      ??I?(J)=??.?X+??IS(J)
      ???I??(J)=??.??P?I???(J))
   ?? CO??I?U?
      CALL ??TI?U??A?X(??J?=L)
      ?? I?? ??. I ?? ?X
  1 ?? Y??(?) =SU?T?X ?.??(K)??2+X??? (K)??2}
      ??I?(?,?)
      ??I?(?,??)
      ??I?E(?,??? XA??(K)? ?A? E?,?)?)
C  ?EA??? ??S?H??? ?E????S.
      TA?U?=??H?C??/?.??A??K?
      ?A?S=??J??2.
      ?A?I?=?.?(??N?
      ?T?US=I?T(?AUS)?
      ???O?=?.I(T?J?)
      ????=??????
      ???=??TA??
      ??P?=???
      ?? I?? K=?,?A?
      IF(( XC??(K?.?L.?I??T?.??T?(K.????)I? ?I??= XC?(K)
      IF(( ?CP?(K).?L.?I??I?).??L?.(K??????)) ?I??= X?P(K)
      IF(( X?P(K).??.???I?T?.???.(K????.??)? ???=?.?
      IF(( ?C??(?).?L.?I??I?.A??.(?? ????)) ???=??T
  ??? ?O??I?U?
      ???=I?????
C  ?? ??I?? THE P?AK ?F THE ?COMPRESSED ?U?SE IT DAT WILL AL?AYS OCCUR
C  ?? ??=I??? FOR ??? ?I?????A??I?? P?O??CT.
```

```fortran
C  XGP IS NOW STORED IN THE XRMW ARRAY (DEFINED..NO EQUIVALENCE)
      DO 110 K=MIN,MAX
      KS=K+IS
      KS=KS-((KS-1)/MAX)*MAX
      XRMW(KS)=-20.*ALOG10(PIPT/XGP(K))
      IF (XRMW(KS).LT.-60.) XRMW(KS)=-60.
110   CONTINUE
      WRITE(5,3)
      WRITE(5,5)
      WRITE(5,1) (XRMW(KK), KK=MIN,MAX)
      DO 120 J=MIN,MAX
120   FREQ(J)=DELF*FLOAT(J-1)
      DO 130 K=MIN,MAX
130   TIME(K)=DELT*FLOAT(K-1)
      WRITE (6,14)
      CALL FRAMEV(0)
      CALL PITE2V(50,800,1020,90,2,23,-1,23HCHARLES KISS = 876-4061,N
     1F)
      CALL JOLK3V(-1.42,FLOX,FLOY,-1024,FREQ(256),APHIS(256))
      CALL JOLK3V(-1.42,FLOX,FLOY,-1024,FREQ(256),XRMW(256))
      CALL JOLK3V(-1.42,FLOX,FLOY,-1024,FREQ(256),APHIMW(256))
      CALL JOLK3V(-1.42,FLOX,FLOY,-400,TIME(825),XRMW(825))
      CALL JOLK3V(-1.42,FLOX,FLOY,-2048,TIME(1),XRMW(1))
      CALL FRAMEV(0)
      CALL PITE2V(50,800,1020,90,2,23,-1,23HCHARLES KISS - 876 4061,N
     1F)
      STOP
      END
```

```fortran
      SUBROUTINE APHASE (ABLANK)
      DIMENSION ABLANK(1)
      MIN=1
      MAX=2048
      TPI=2.*3.14159265
      DO 1 K=MIN,MAX
      ANUM=ABLANK(J)/TPI
      NUM=INT(ANUM)
      BNUM=FLOAT(NUM)
      ABLANK=ANUM-BNUM
      ABLANK(J)=ABLANK*TPI
1     CONTINUE
      RETURN
      END
```

```fortran
      SUBROUTINE FFT(XR,XI,L)
      DIMENSION XR(1),XI(1),KD(16),KND(16)
      INTEGER HALFN,R,S
      DOUBLE PRECISION TPI
      TPI=6.283185307179586
      UNIT=1.
      M=IABS(L)
      IF(M.ET.0) RETURN
      HALFN=2**(M-1)
      N=HALFN+HALFN
      IF(M.EQ.1) GO TO 3
      IF(L.ST.GT GO TO 1
      TPI=-TPI
      UNIT=-UNIT
    1 CONTINUE
      FI=TPI/FLOAT(N)
    3 S=N
      INC=1
      DO 2 I=1,M
      R=S
      S=S/2
      KTWO=1
      DO 5 J=1,S
      ANG=FLOAT(KTWO-1)*FI
      TR=COS(ANG)
      TI=SQRT(1.-TR**2)*UNIT
      KTWO=KTWO+INC
      KLIM=J+S-R
      DO 5 K=J,KLIM,R
      KANDS=K+S
      HOLDR=XR(K)-XR(KANDS)
      HOLDI=XI(K)-XI(KANDS)
      XR(K)=XR(K)+XR(KANDS)
      XI(K)=XI(K)+XI(KANDS)
      XR(KANDS)=TR*HOLDR-TI*HOLDI
      XI(KANDS)=TI*HOLDR+TR*HOLDI
    5 CONTINUE
      INC=INC+INC
    2 CONTINUE
      IF(L.GT.0) GO TO 4
      HOLDR=1./FLOAT(N)
      DO 10 J=1,N
      XR(J)=XR(J)*HOLDR
   10 XI(J)=XI(J)*HOLDR
C UNSCRAMBLE THE TRANSFORM
    4 IF(M.EQ.1) RETURN
      DO 6 J=1,M
      KD(J)=0
    6 KND(J)=1
      DO 7 KTO=1,N
      IHP=1
      DO 8 J1=1,M
      IF((KAD+KND(J1)).NE.IHP) GO TO 8
      KND(J1)=KD
      KD(J1)=1-KD(J1)
```

```
8   IHP=IHP+IHP
    LOC=T
    S=N*LF
    DO 7 L=1,N
    LOC=LOC+ND(L)*5
4   S=S/2
    IF(KE).GE.LOC)  GO TO 7
    HOLDR=XR(KAD)
    XR(KAD)=XR(LOC)
    XR(LOC)=HOLDR
    HOLDI=XI(KAD)
    XI(KAD)=XI(LOC)
    XI(LOC)=HOLDI
7   CONTINUE
    RETURN
    END
```

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